Sub-diffusion, cage effects and collective re-arrangements in granular media

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Introduction

Various experiments on vibrated granular media indicate a possible analogy between glass and granular media:

- **Strength**: the relation between thermal and a-thermal systems.
- **Weakness**: only at the macroscopic level (slow compaction experiment: Chicago and Rennes groups) or at the thermodynamical level (T., Díaz et al.)

For glass forming systems, sub-diffusion and slow relaxation have been associated with “cage effects” and “spatially heterogeneous dynamics.”

- Molecular Dynamics Simulations of hard spheres and Lennard-Jones liquids provide a lot of data on the underlying microscopic mechanisms (Glotzer et al.).
- Colloidal Suspensions Experiments at high density + confocal microscopy provide direct observation of the individual particle paths (Watts et al.).

Here:

- Experimental study of the diffusion properties and microscopic behaviors in a granular media, driven as differently as possible from thermal excitation.
- Can we give a precise meaning to the above analogy?
The experimental set up

- A 2D bi-disperse dry granular media under cyclic shear (not just an analog computer)

The system
- 8000 particles
- Bi-disperse (Ø=4 and 5mm)
- Quasi-static shear
- Constant Volume ($\Phi=0.86$)

The protocol
- 10,000 cycles
- $\theta_{max}=10^\circ$, max strain=0.3
- 500 tracers are followed
- A snapshot is taken at each cycle

NB: Different from an analog computer (good models of friction are still lacking)
Typical trajectory

NOTA BENE
- Typically the same size relatively to the particle diameter as in D. Weitz experiments
- Smaller than in S. Glotzer numerical simulations

Hard spheres vs. "soft" potentials
Intermittent moves and subdiffusion

- Fat tails, Intermittency
- Non-Gaussianity factor depends on the timescale
- Crossover between subdiffusive and diffusive motion at $r^* \approx 0.3$ and $t^* \approx 300$

\[ P(\Delta X(\tau)/\sigma) \]

\[ \frac{\langle \Delta r^2(\tau) \rangle}{\langle \Delta r^2(\tau) \rangle}^{1/2} \]
Anti-correlated moves

\[
\langle y_{12} \rangle = 0 \quad \langle x_{12} \rangle < 0
\]

for \( r_{01} < r^* \)

\[
\langle x_{12} \rangle = c(\tau) r_{01}
\]

for \( r_{01} > r^* \)

At a given timescale \( \tau \):

\[ P(x_{12} \mid r_{01}; \tau) \] and \[ P(y_{12} \mid r_{01}; \tau) \]
Anti-correlations (II)

Varying the timescale $\tau$:

- $\forall \tau < t^*$, the saturation occurs for $r_{01} = r^*$
- For $\tau > t^*$, the anti-correlations vanishes
For \( \tau < \tau^* \), \( \text{rms}(x_{12}) \) increases with \( r_{01} \), propensity to move is larger for previously rapidly moving particles.

While \( \text{rms}(y_{12}) \) remains constant, preferentially parallel to the previous move.

\[ \Rightarrow \text{suggest the string-like cooperation observed by Donati et al.} \]
Heterogeneities (II)

\[ F_2(t) = \langle \cos(qr_{01}) \rangle; \quad F_2(2t) = \langle \cos(qr_{02}) \rangle; \]
\[ F_3(t,t) = \langle \cos(qr_{12}) \cos(qr_{01}) \rangle; \]

Purely Homogeneous Dynamics: \( F_3(t,t) = F_2(t)^2 \)
Purely Heterogeneous Dynamics: \( F_3(t,t) = F_2(2t) \)

The dynamics has a significant heterogeneous part (already in the \( \beta \)-regime)

\[ q = 1/r^* \]
Cages and collective dynamics

- Cages are rather small ($r^* = 0.3$)
- What are cages?
- How many grains are involved in a cage re-arrangement?

⇒ Need to follow all particles
⇒ New exp. set up
Some information on the structure

- A rather long ranged structure
- Significant fluctuations in the local density
- Work under progress: spatio-temporal structure
Direct observation of the dynamics
A closer look at the grain scale

Void redistribution allows cage re-arrangement

Dynamics facilitation (but also inhibition)

How long is the range of the correlations?
A broader look at the dynamics
First quantitative estimations

\[ T_i(r) : \text{time for grain } i \text{ to reach the circle of radius } r \]

\[ r_i \]

\[ T_{i,l}(r) = \frac{<T_{i,l} - T_{av}>^2}{<T_{i,l} - T_{av}>^2} \]

\[ m_2(l) = \frac{<T_{i,l} - T_{av}>^2}{<T_{i,l} - T_{av}>^2} \]

\[ m_2(l) \]

\[ L \]

The correlation length is maximum at the scale of the cage re-arrangements up to 7 particle diameters.

Hurley and Harrowell
Conclusion and Perspectives

- Dense granular media are analogous to glasses in the sense that their diffusion properties are identical at timescales larger than the thermal regime.

- It is a good idea to make use of theoretical ideas from glasses in the field of dense granular media.

- A granular experimental set-up is an efficient tool to study glasses at the particle level.

Further work will deal with:

- A more precise study of the microscopic dynamics (Clusters?, Strings?, Dynamical heterogeneities?, \( \chi^4 \)).

- The study of a response function (RFD, \( T_{eff} \)).

- Aging with or without compaction.