О милых спутниках, которые наш свет
Своим сопутствием для нас животворили,
Не говори с тоской: их нет,
Но с благодарностию: были.

В. А. Жуковский «Воспоминание» (1827)

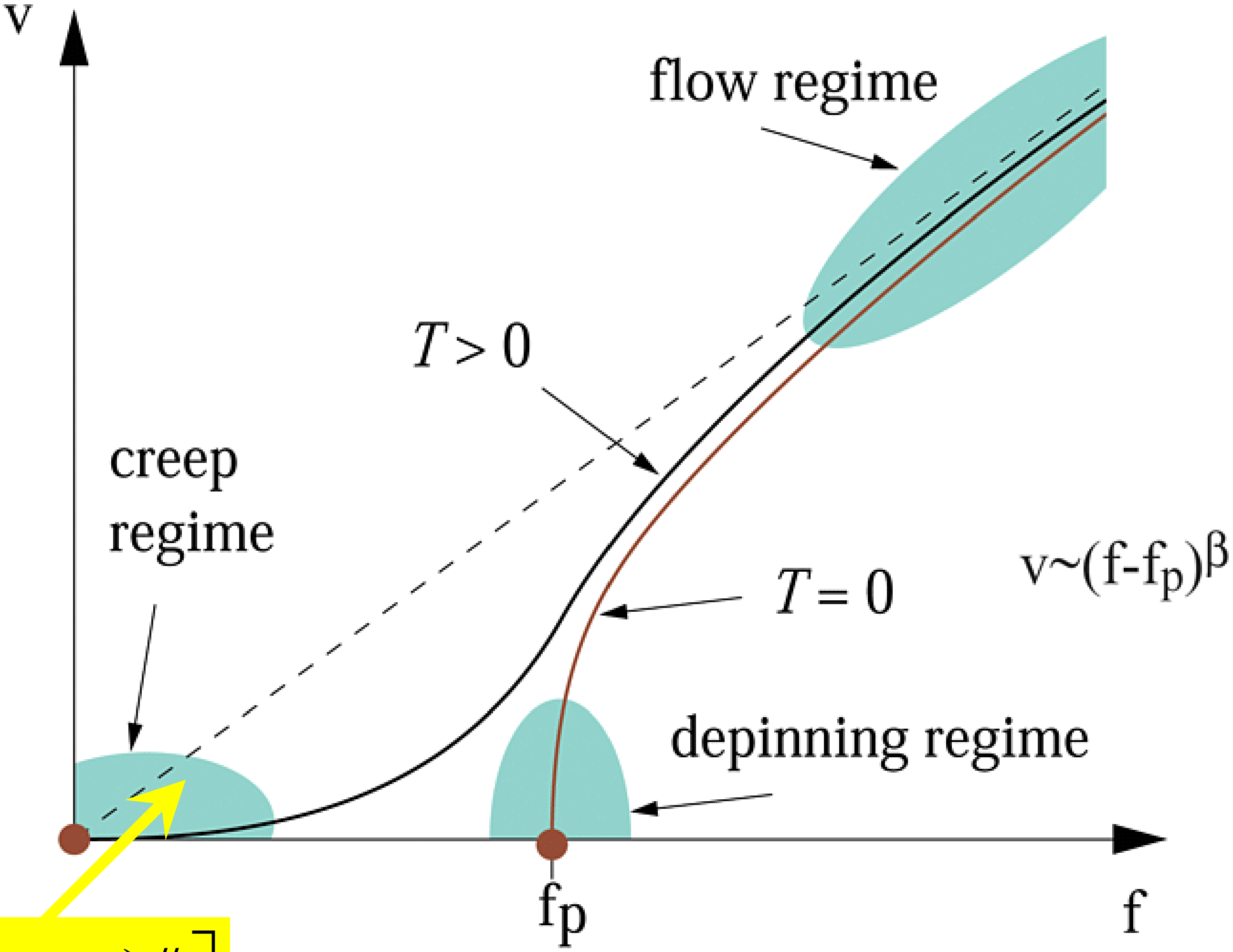
Des camarades chéris, qui notre monde
Ensoleillaient de leur société
Ne dis pas, angoissé, "ils ne sont plus"
Dis, bénissant, "ils existaient."

Day in memory of Miguel Ocio on Monday, October 3rd 2005

On the glassy behavior of vortex systems

V. Vinokur

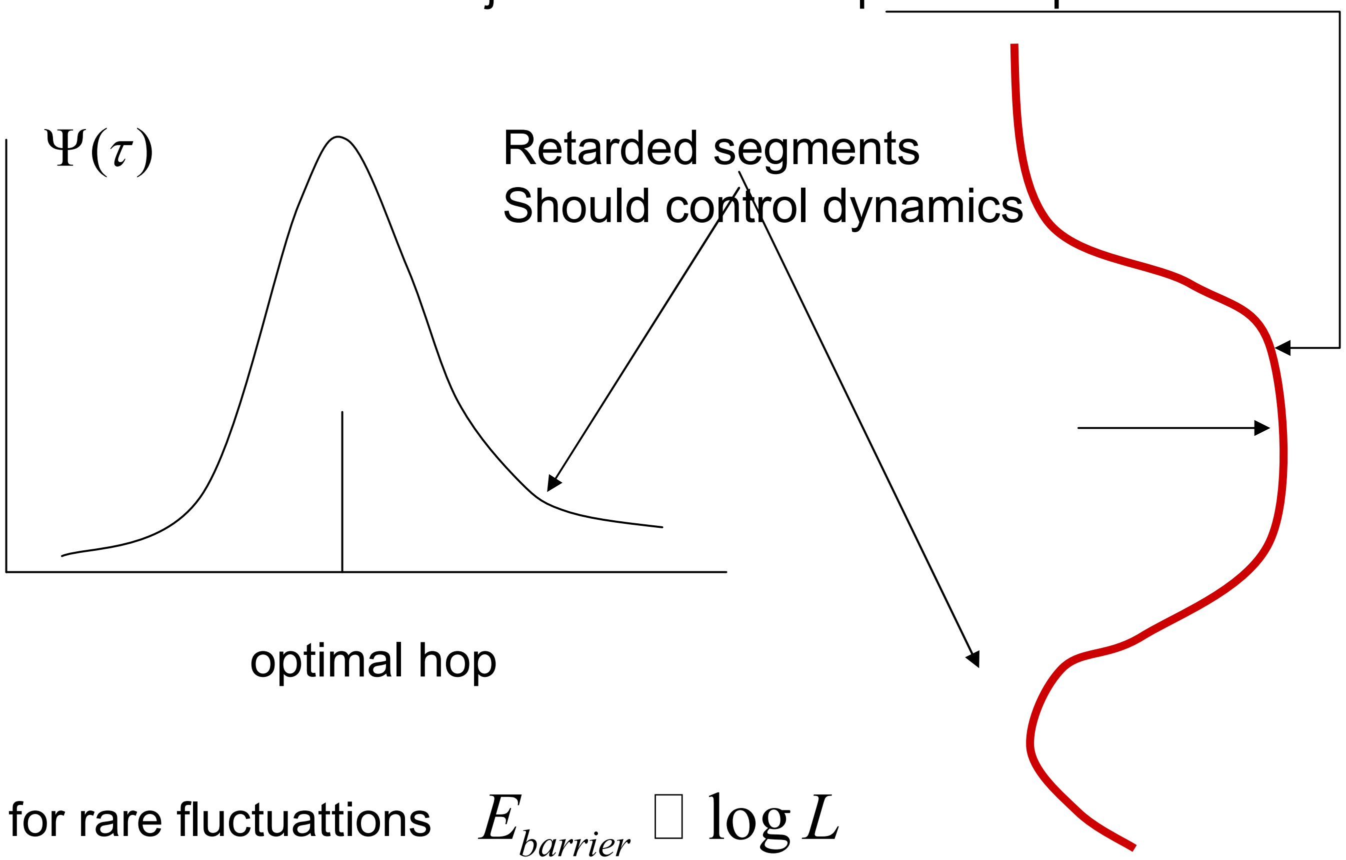
Generic I-V characteristic of the driven vortex lattice



$$v \sim \exp\left[-\frac{E_p}{T}\left(\frac{f_p(T)}{f}\right)^{\mu}\right]$$

How can it work?

How can our object choose this optimal hop?



optimal hop

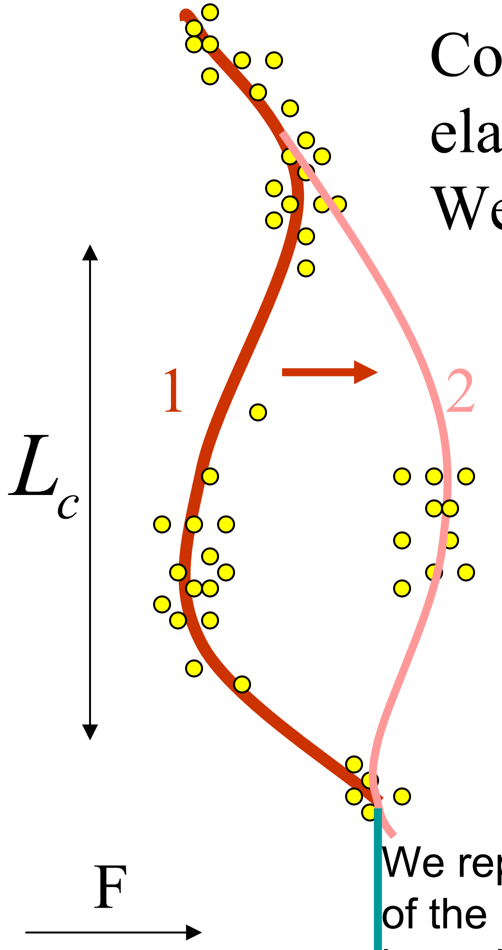
But… for rare fluctuattions $E_{barrier} \square \log L$

(at best)

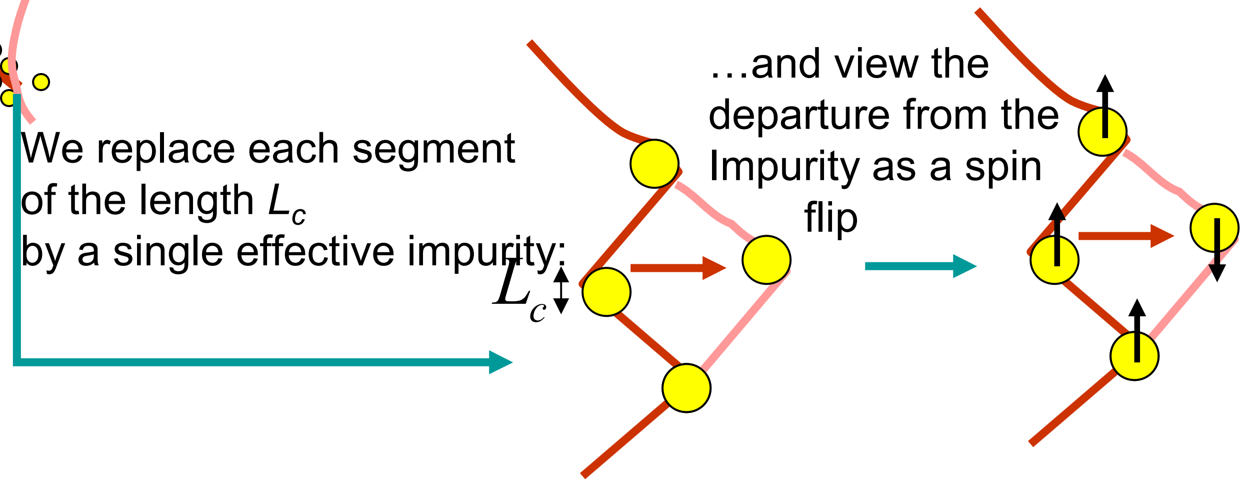
Energy gain due to external force $\square\ L$

$\Psi(\tau)$?

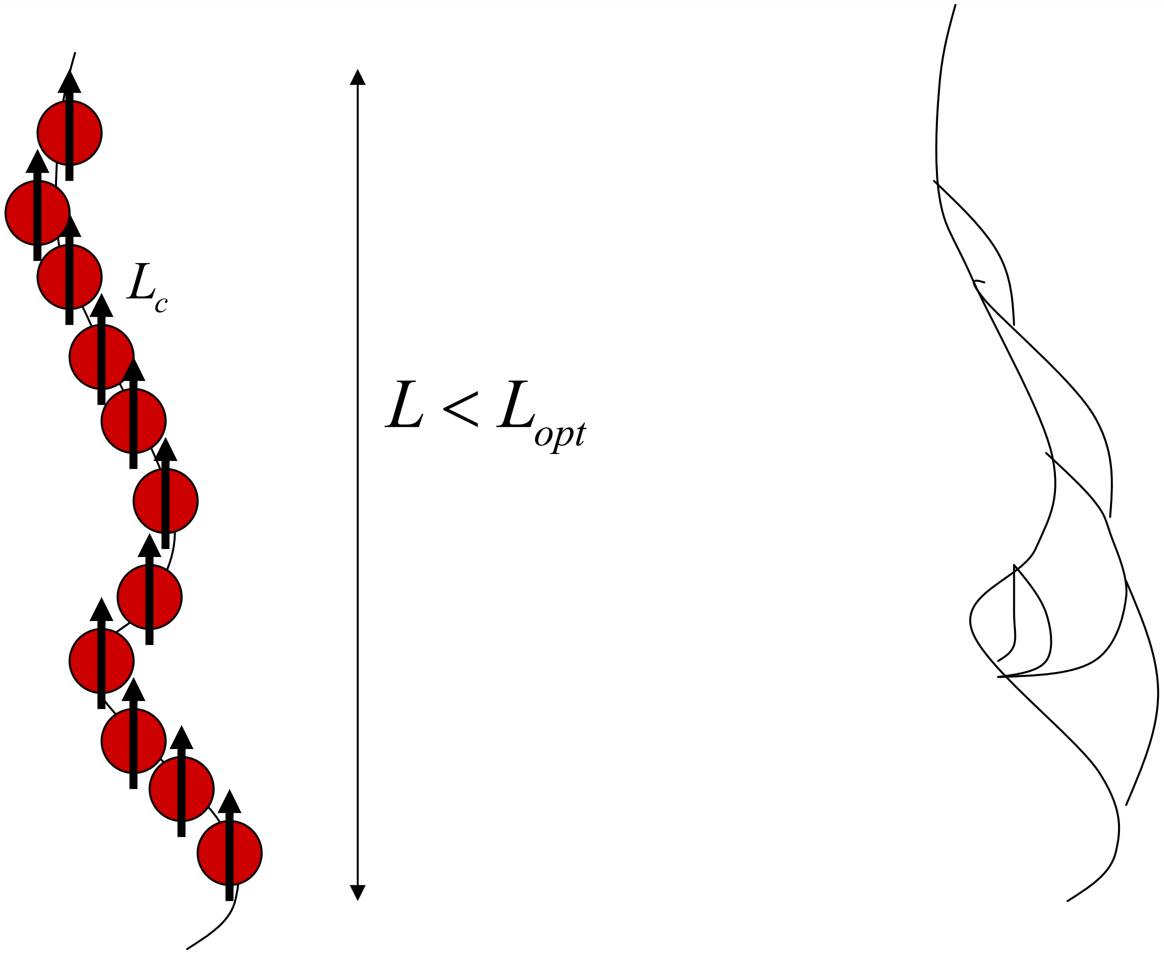
Consider a change in the configuration of the elastic string in the random field:
We call it departure from the initial position 1.

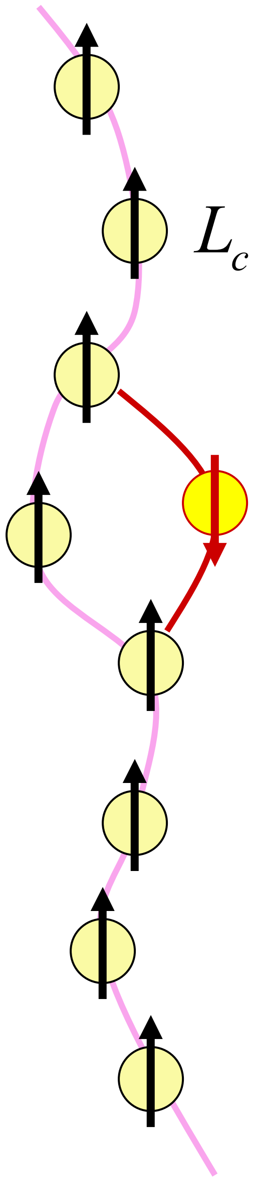


We are interested in the distribution density $\Psi(\tau)$ of the departure times τ. To find it we will determine first the probability distribution $W(E)$ of barriers E controlling string dynamics

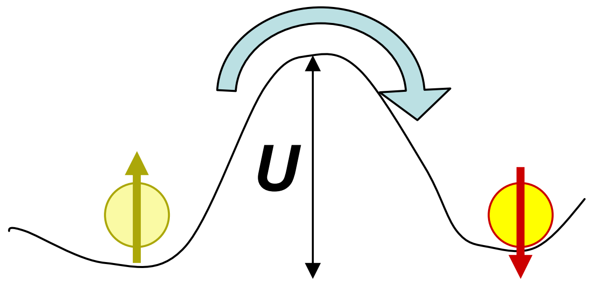


To derive the distribution of energy barriers we define the elementary moving "units" of the string as segments of length L_c. The barriers controlling the hop of these units to the nearest metastable state fluctuate about U_c.



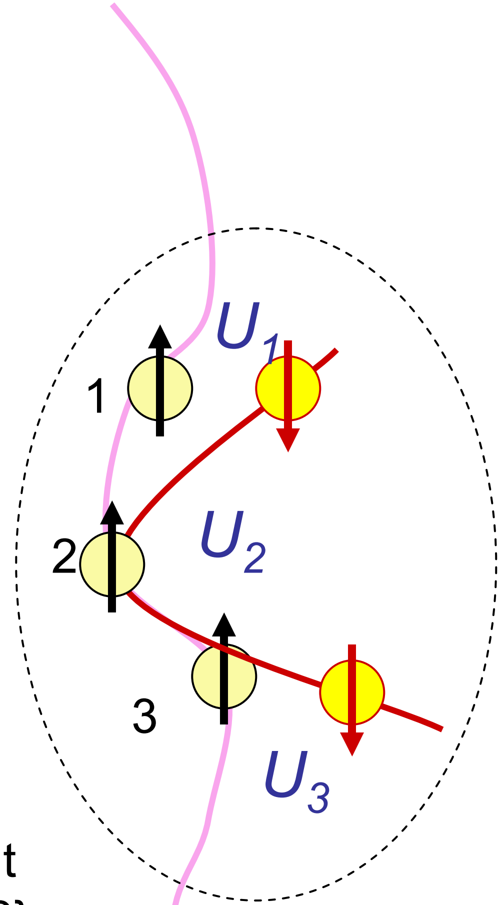


It costs energy U to flip (depart from) a site



The barriers controlling the flips (hops) to the neighboring site fluctuate about U_c.

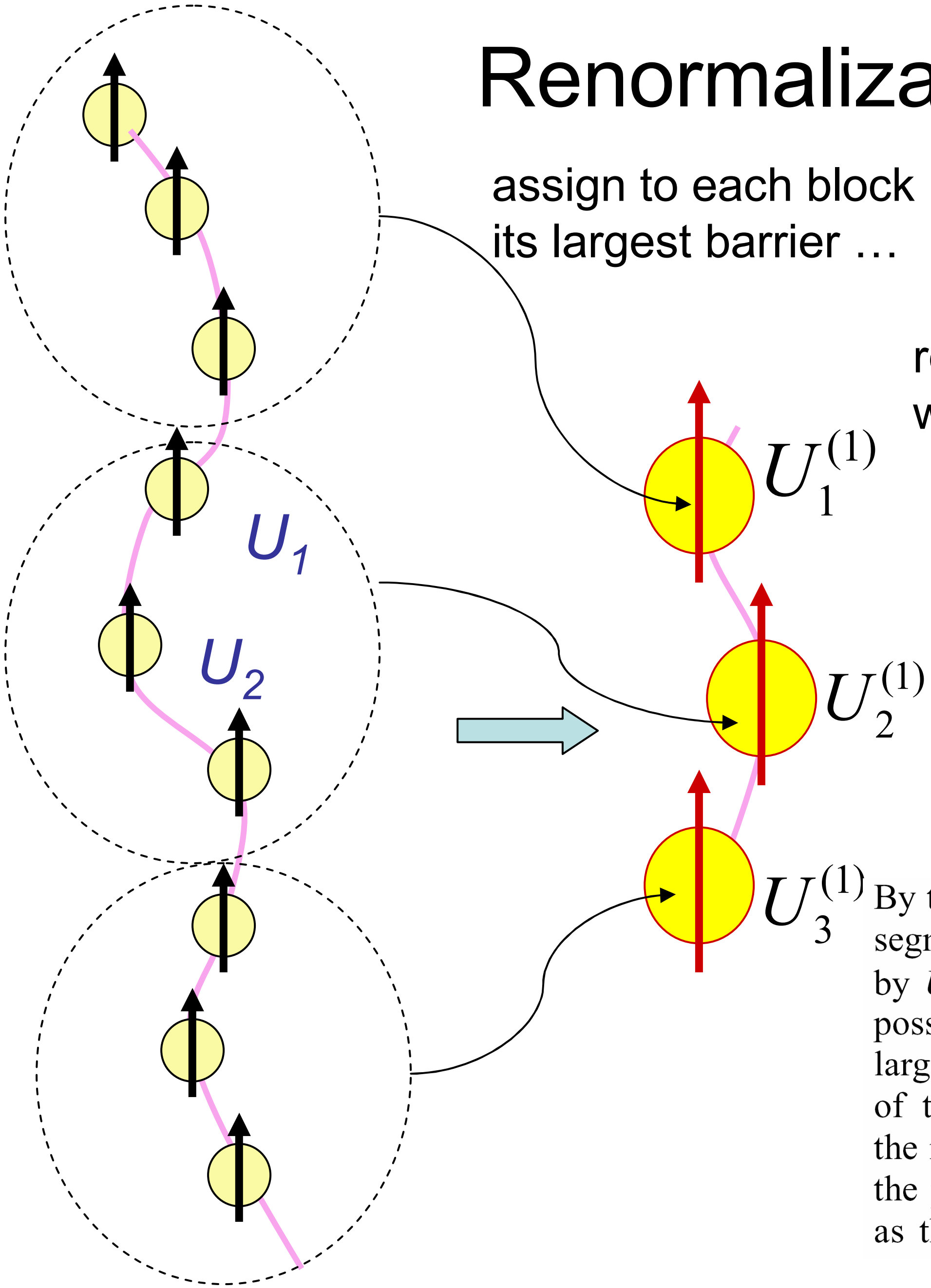
Consider a segment of three units: {1,2,3}



Departure of the segment is controlled by $U^{(1)}=max\{U_1,U_2,U_3\}=U_2$

Renormalization procedure

assign to each block
its largest barrier …



replace each block by a new unit
with these new barriers…

…and repeat the procedure…

Generally we consider blocks
consisting of *m* units

By the same argument as above, we find that the hop of a segment L' composed of m blocks L_1 is again controlled by $U' = \max\{U_j^{(1)}\}$. Repeating this procedure, we list all possible configurations of the advancing string L. If for large m a limiting form exists for the distribution function of the energy barriers, this form must be stable under the max operation. In other words if $U = \max\{U_i\}$, then the probability distribution of the extrema U is the same as that of each member U_i of the set.

The probability $\mathcal{P}_L(z)$ that the largest energy barrier controlling the hop of a string of length L is less than U is given by the solution of the functional equation

$$\mathcal{P}_L(z) = \left[\mathcal{P}_L(a_n z + b_n)\right]^n,$$

where $z = (U/U_c - b_n)/a_n$ and all lengths are measured in units of L_c.

The probability distribution of the largest energy barriers for a pinned segment of length L is then,

$$\mathcal{P}_L(U) \sim \exp\big[-e^{-(U-U_c \ln(L/L_c)/U_c}\big] \\ = \exp\big[-(L/L_c)e^{-U/U_c}\big]$$

and the corresponding probability density is given by

$$p_L(U) = \frac{d\mathcal{P}_L}{dU} \sim \frac{L}{L_c} e^{-U/U_c} \exp\big[-(L/L_c)e^{-U/U_c}\big].$$

Global distribution: $$W(U) = \int_{L_c}^{\infty} dL\, n_L \mathcal{P}_L(U)$$

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Elastic String in a Random Potential

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(Received 3 January 1992)

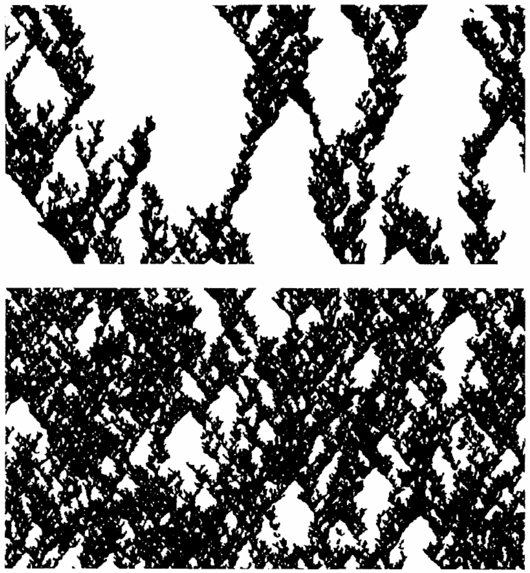


FIG. 2. Maps of string velocity for $f = 0.01$ (top) and $f = 0.076$ (bottom) and $F_p = 0.1$. The vertical axis is time, while the horizontal axis is the position z on the string. Dark regions indicate where the velocity exceeds 0.01. Maps are shown for strings of size $L = 4096$ evolving over a time interval $\Delta t = 30$.

n_L : the density of pinned segments of the length *L*

$$n_L \sim 1/L^{\nu} \qquad \nu = 1 + d/d_f > 2$$

$$\boxed{W(U) \sim e^{-U(\nu-1)/U_c}}$$

$$\tau = \tau_0 \exp(U/T) \qquad \Psi(\tau)d\tau = W(U)dU,$$

$$\boxed{\Psi(\tau) \sim T(\tau_0/\tau)^{1+\alpha}, \;\; \alpha = (\nu - 1)T/U_c}$$

The mean motion is controlled by the largest departure/waiting time corresponding to the hop of the optimal segment $L_{opt}(F)$.

segments on scales $L > L_{\rm opt}$ slide freely. $\Rightarrow$

$$\langle \tau \rangle \sim \int^{\tau_{\rm max}} d\tau \Psi(\tau)\tau \sim \exp[(1 - \alpha)U(F)/T]$$

$$\tau_{\rm max} = \tau_0 \exp[U(F)/T]$$

This is by construction the time over which $L_{\rm opt}$ advances a transverse distance $u_{\rm opt}$. The mean velocity is then

$$\boxed{v \simeq u_{\rm opt}/\langle \tau \rangle \simeq \exp[-(1 - \alpha)U(F)/T]}$$

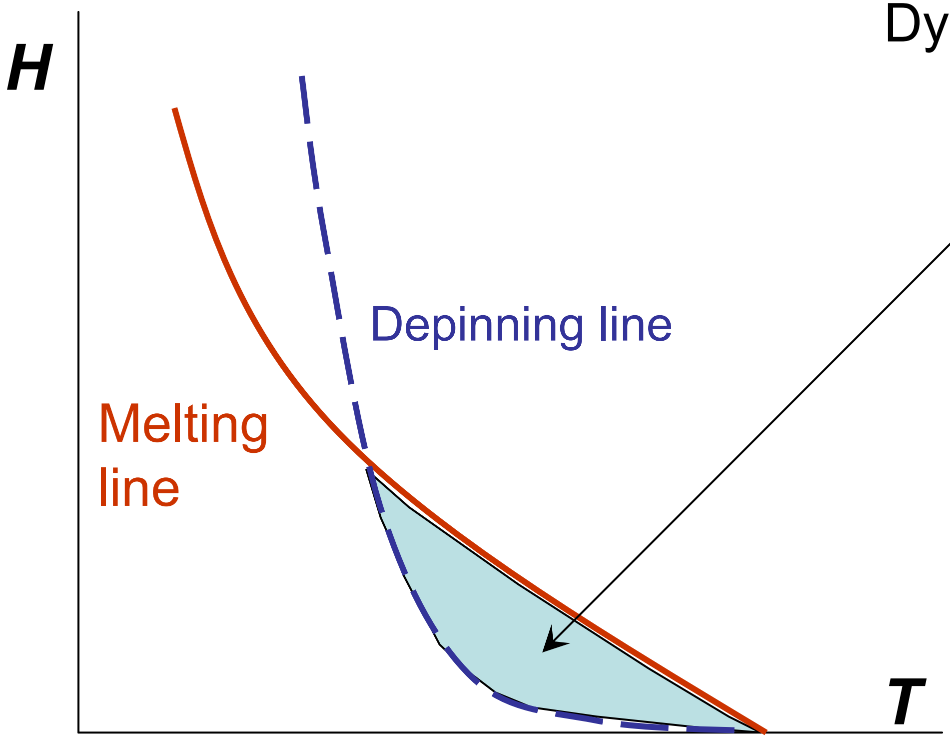
Depinning temperature: T_{dp}

$$T > T_{dp} \ : \ J_c \propto \exp(-T / T_{dp})$$

$$\alpha = 1$$

Dynamics becomes marginal:

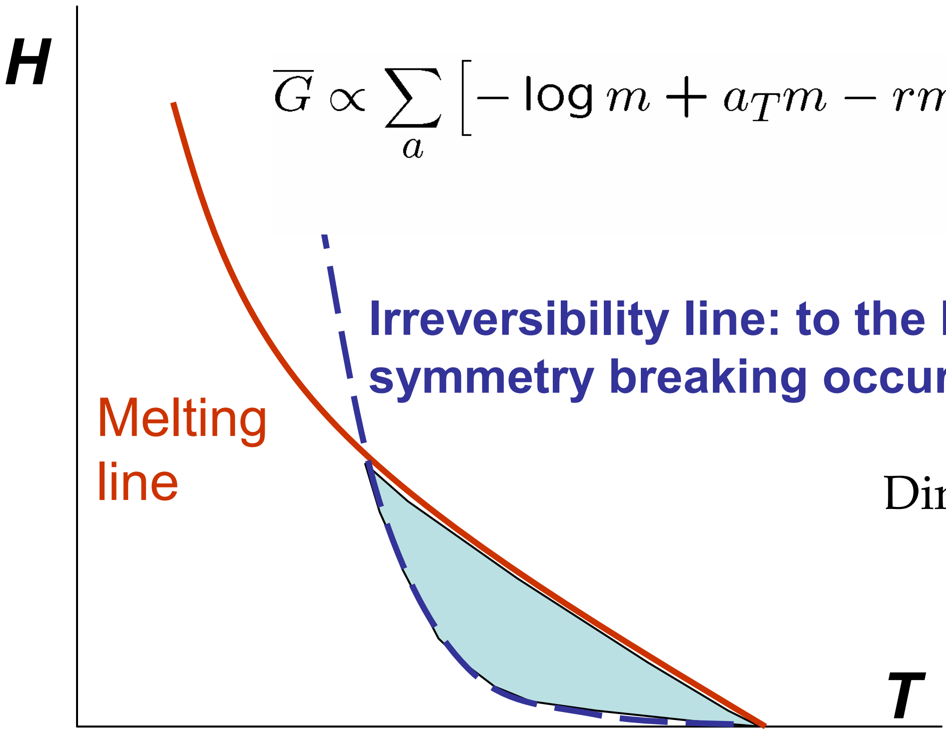
$$v \propto f^{2+\mu}$$



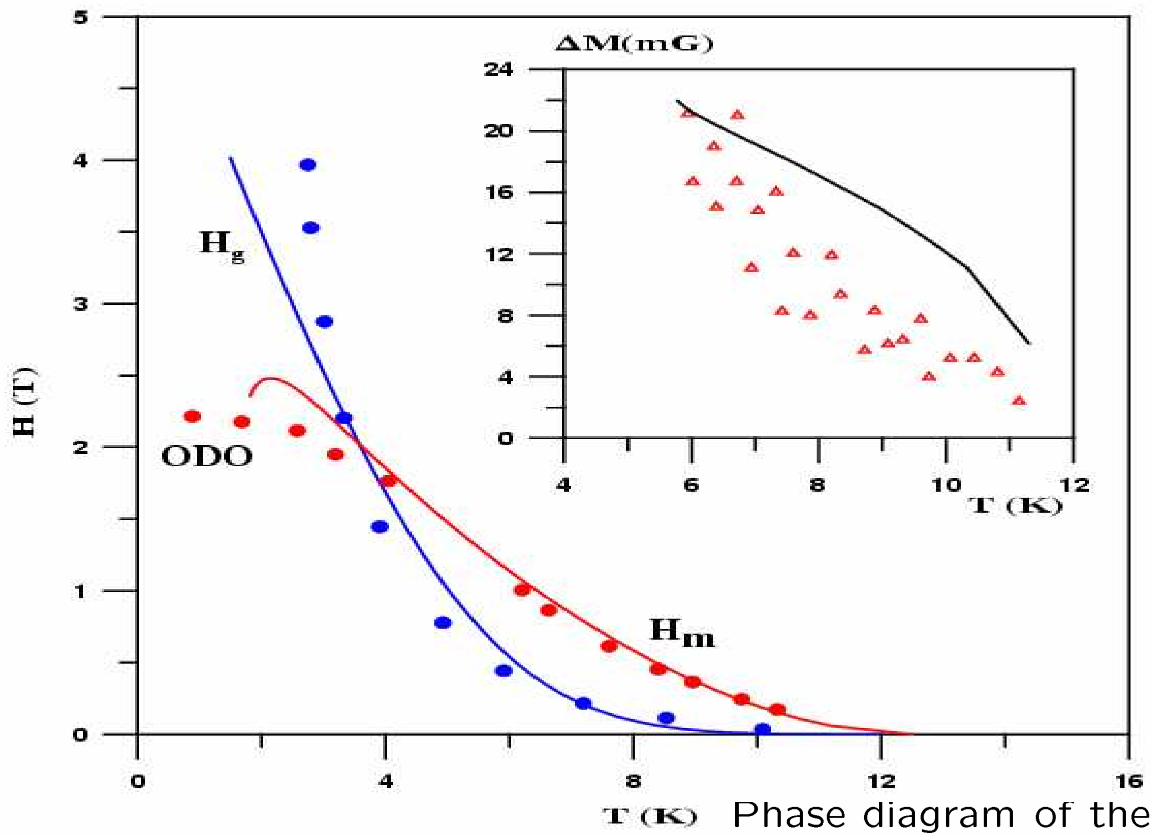
$$G = \int_{\mathbf{x}} \left[\left| \frac{\hbar^2}{2m^*} \left(\nabla + \frac{2ie}{\hbar c} \mathbf{A} \right) \psi \right|^2 + \alpha \left(T - T_c \right) \psi^* \psi + \frac{\beta}{2} \left(\psi^* \psi \right)^2 + \frac{(\mathbf{B} - \mathbf{H})^2}{8\pi} \right],$$

Expanding $\psi(x)$ in the basis of the LLL wave-functions, the Gaussian effective free energy can be expressed via the variational matrix parameters $m_{ab} = \langle \psi_a^* (k) \, \psi_b (-k) \rangle$,

$$\overline{G} \propto \sum_a \left[-\log m + a_T m - r m^2 - q (\widetilde{m})^2 m^2 \right]_{aa} - \frac{q}{4} \sum_{a,b} m_{ab}^4,$$



Dingping Li, B. Rosenstein (2005)



Phase diagram of the organic superconductor $\kappa-(BEDT-TTF)_2Cu\left[N\left(CN\right)_2\right]Br$. Comparison of the theoretical melting line (red) and the glass line (blue) with the experimental data from T. Shibauchi, M. Sato, S. Ooi, and T. Tamegai, Phys. Rev. **B57**, 5622 (1998) shown by red and blue points respectively. Shown in the inset is the calculated magnetization jump at melting and the corresponding experimental data

Conclusions:

The low-temperature dynamics of driven elastic manifolds in random environment is governed by a power-law distribution of hopping times.

It looks like the slow dynamics of elastic systems controlled by the extreme value statistics of energy barriers and the replica symmetry breaking description of disorder within the field-theoretical Ginzburg-Landau approach are in fact the two facets of the same phenomenon.