Spinful Majorana fermions and magneto-electricity in 1D quantum-wire/superconductor heterostructures

Panagiotis Kotetes

Collaborators: Alexander Shnirman and Gerd Schön

NanoCTM workshop, Corsica 2012

Karlsruhe Institute of Technology

A sneak peek...

- **Introduction**:

  Topological quantum computing: the quest for Majorana fermions

  Majorana fermions based on semiconducting wires

  What is the nature of the emerging topological phase?

- **Interaction of Majorana fermions in TNT and NTN junctions (T $\rightarrow$ topological, N $\rightarrow$ non-topological)**

  1. The Majorana Josephson current can be “magnetically” tuned

  2. Control of a spin current by a superconducting phase

  3. Alternative pathways for topological quantum computation **(TQC)** and Majorana fermion braiding

Topological vs non-topological qubits

- Non-topological qubits are vulnerable to noise and decoherence

  1. Superconducting qubits suffer from noise e.g. 1/f

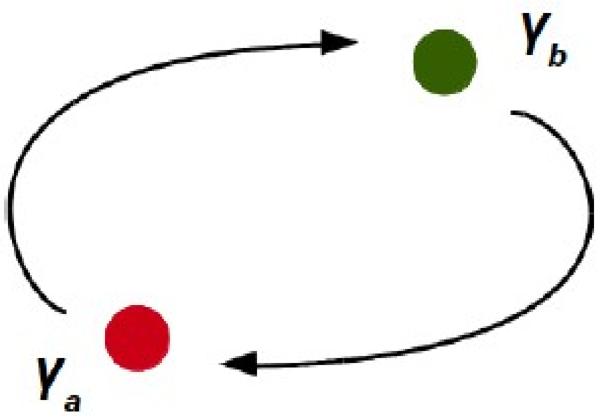
  2. Spin qubits in quantum dots couple to nuclear spins

- **In contrast**, topological qubits (**TQ**) are immune to decoherence and long-lived

- The protection restricts the accessible operations only to **braiding**

Kitaev, Annals of Phys. (2003) and
Nayak et al., Rev. Mod. Phys. (2008)

Braiding of two Majorana fermions →



Majorana fermions in superconductors

- First important discussions of Majorana fermions were in the frame of p-wave superconductors

VOLUME 86, NUMBER 2 PHYSICAL REVIEW LETTERS 8 JANUARY 2001

Non-Abelian Statistics of Half-Quantum Vortices in *p*-Wave Superconductors

D. A. Ivanov
Institut für Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
(Received 17 May 2000)

Excitation spectrum of a half-quantum vortex in a *p*-wave superconductor contains a zero-energy Majorana fermion. This results in a degeneracy of the ground state of the system of several vortices. From the properties of the solutions to Bogoliubov–de Gennes equations in the vortex core we derive the non-Abelian statistics of vortices identical to that for the Moore-Read (Pfaffian) quantum Hall state.

DOI: 10.1103/PhysRevLett.86.268 PACS numbers: 71.10.Pm, 73.43.–f, 74.90.+n

Majorana fermions in heterostructures

- Experimentally feasible setups were **only recently** proposed

PRL **105,** 177002 (2010) PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2010

Helical Liquids and Majorana Bound States in Quantum Wires

Yuval Oreg,[1] Gil Refael,[2] and Felix von Oppen[3]

[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, 14195 Berlin, Germany*

(Received 16 March 2010; published 20 October 2010)

We show that the combination of spin-orbit coupling with a Zeeman field or strong interactions may lead to the formation of a helical electron liquid in single-channel quantum wires, with spin and velocity perfectly correlated. We argue that zero-energy Majorana bound states are formed in various situations when such wires are situated in proximity to a conventional s-wave superconductor. This occurs when the external magnetic field, the superconducting gap, or, most simply, the chemical potential vary along the wire. These Majorana states do not require the presence of a vortex in the system. Experimental consequences of the helical liquid and the Majorana states are also discussed.

DOI: 10.1103/PhysRevLett.105.177002 PACS numbers: 74.78.Na, 03.67.Lx, 73.63.Nm, 74.78.Fk

The standard topological 1D wire setup

Kinetic term+chemical potential S-O coupling Zeeman field Pairing order parameter

$$\mathcal{H} = \frac{1}{2}\int dk\, \widehat{\Psi}_k^\dagger \left\{ \left[\frac{(\hbar k)^2}{2m} - \mu \right] \tau_z + v\hbar k \tau_z \sigma_y - \frac{g\mu_B}{2} \mathcal{B}_x \tau_z \sigma_x - \Delta e^{-i\varphi\tau_z} \tau_y \sigma_y \right\} \widehat{\Psi}_k$$

BdG 4-component spinor $\widehat{\Psi}_k^\dagger = \left(\begin{array}{cccc} \psi_{k\uparrow}^\dagger & \psi_{k\downarrow}^\dagger & \psi_{-k\uparrow} & \psi_{-k\downarrow} \end{array} \right)$

Only the inherent particle – hole symmetry persists $\tau_x \mathcal{H}_{-k}^* \tau_x = -\mathcal{H}_k$

Class D **CRE** topological superconductor
→ $\mathbf{Z}_2$ invariant in 1D → ***Majoranas***

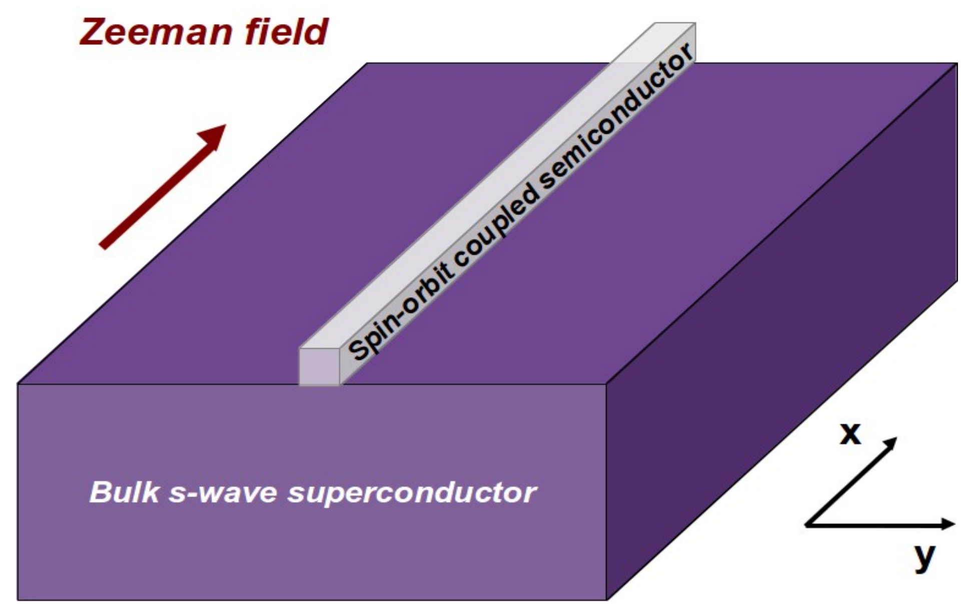
Altland and Zirnbauer, PRB (1997),

Dahlhaus, Béri and Beenakker, PRB (2010)

Kane and Mele, PRL (2005),

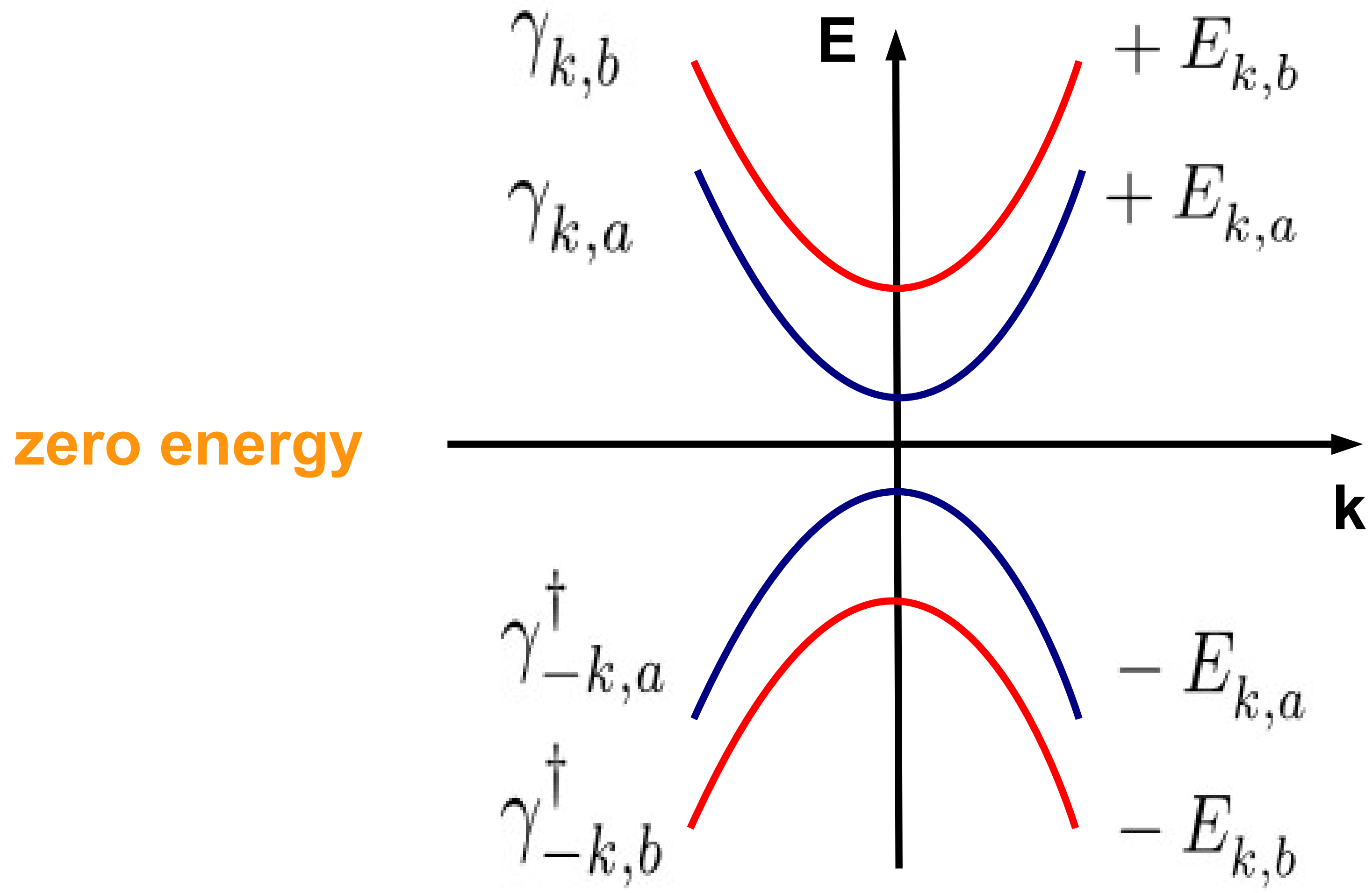
Qi et al., PRB (2008),

Ryu et al., New J. Phys. (2010)



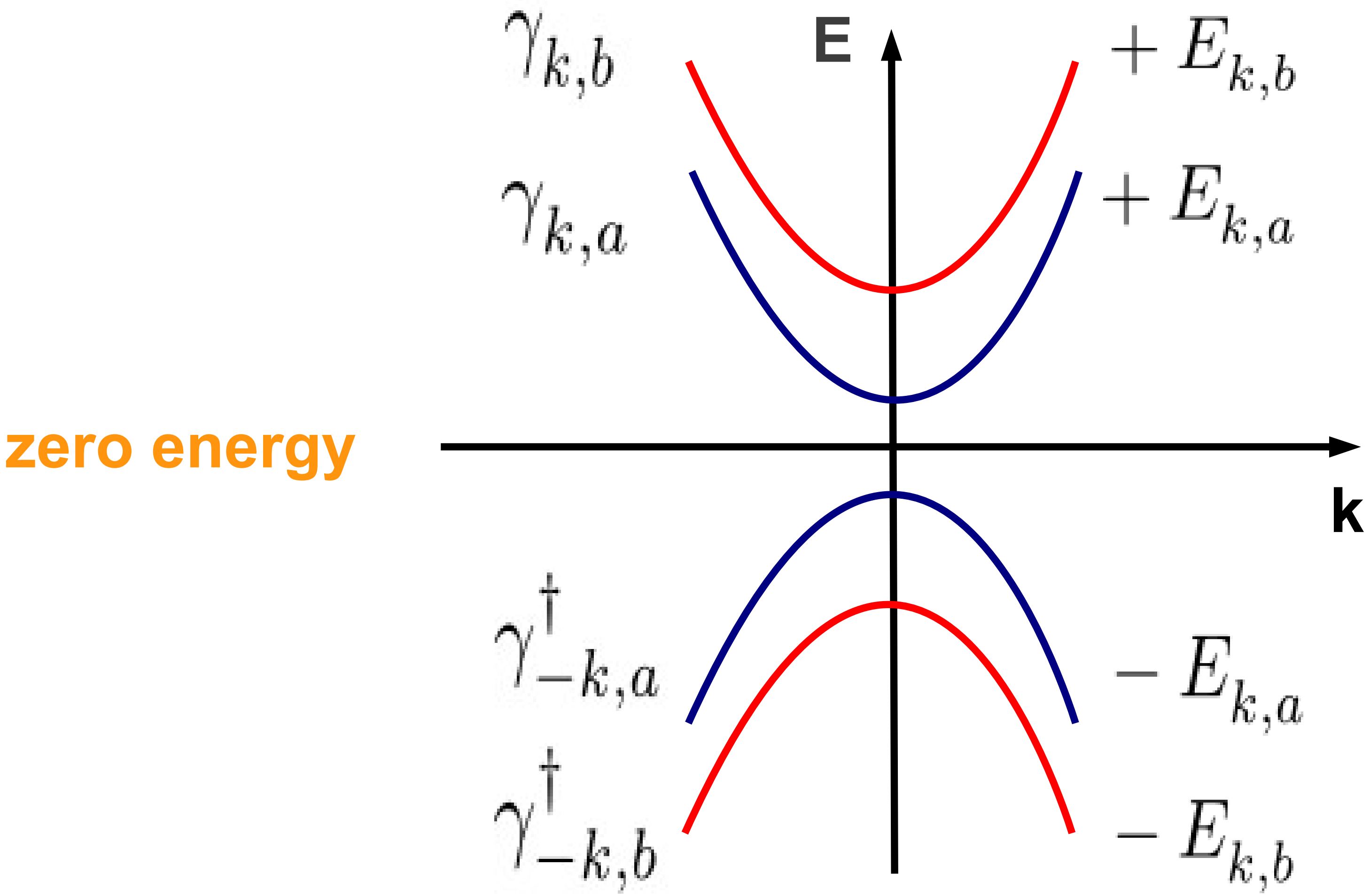
Majorana fermions in an infinite system

- **p-h** symmetry constraints the Bogoliubov eigenoperators and the energy spectrum



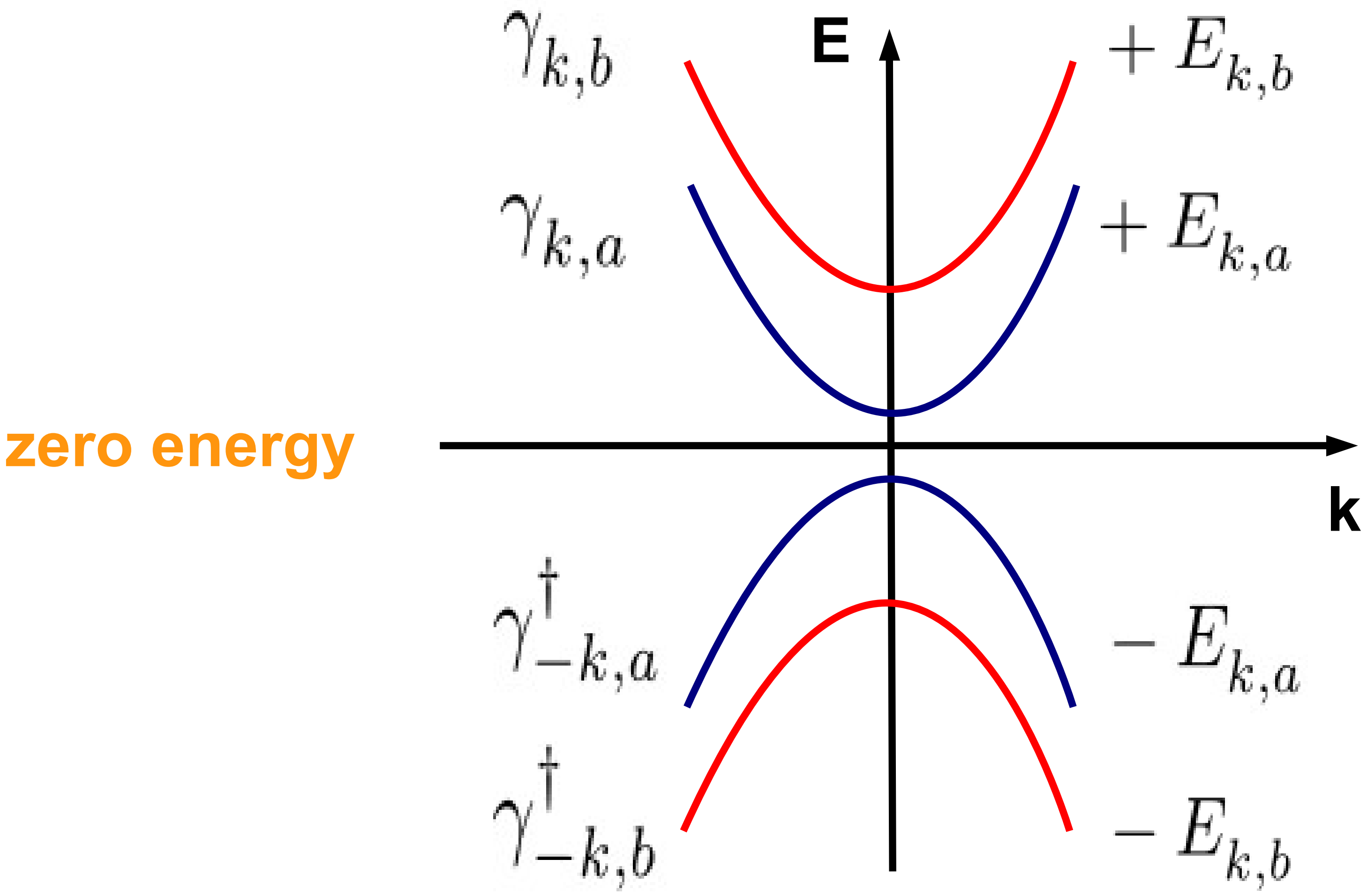
Majorana fermions in an infinite system

- **p-h** symmetry constraints the Bogoliubov eigenoperators and the energy spectrum



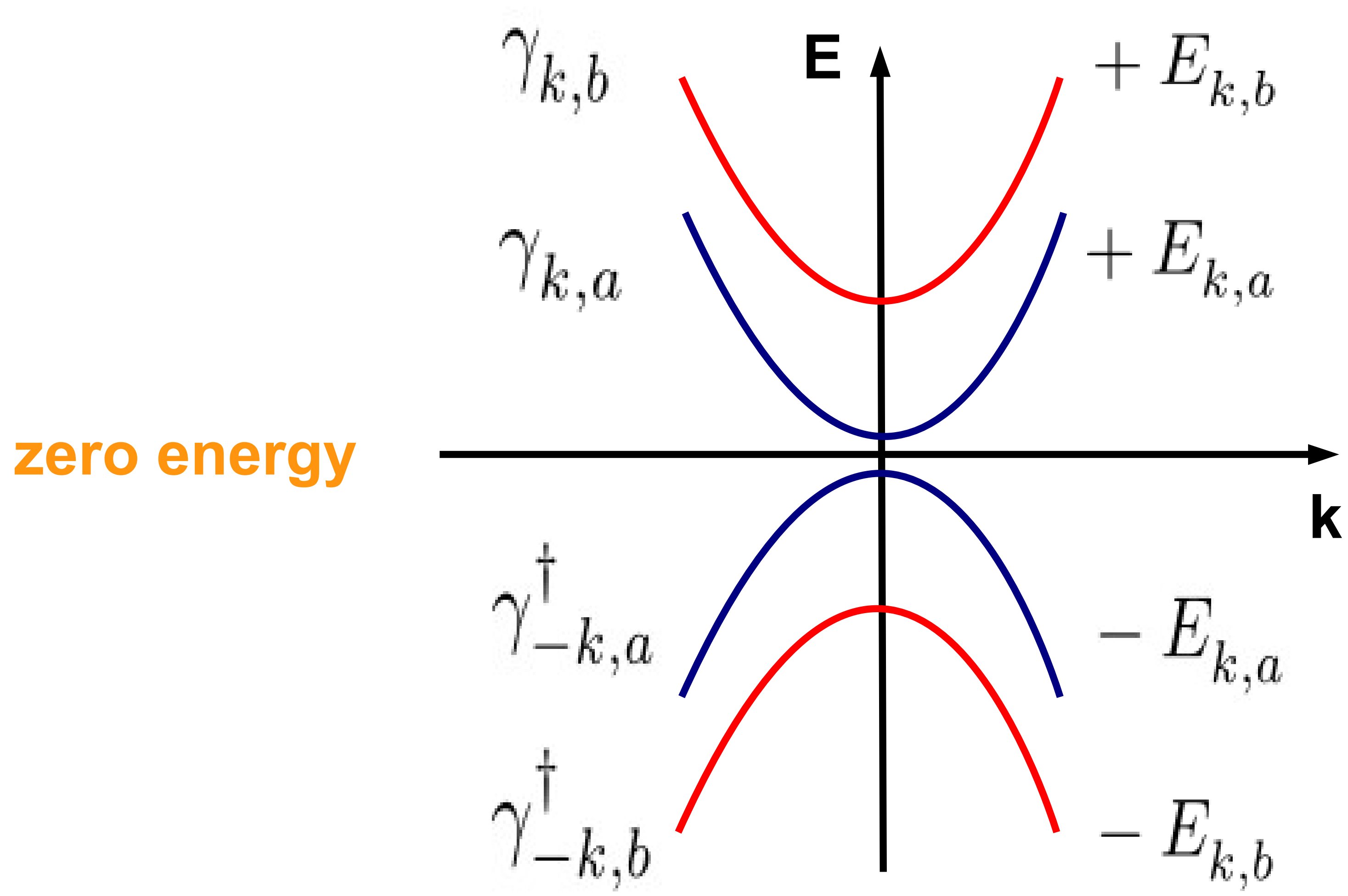
Majorana fermions in an infinite system

- **p-h** symmetry constraints the Bogoliubov eigenoperators and the energy spectrum



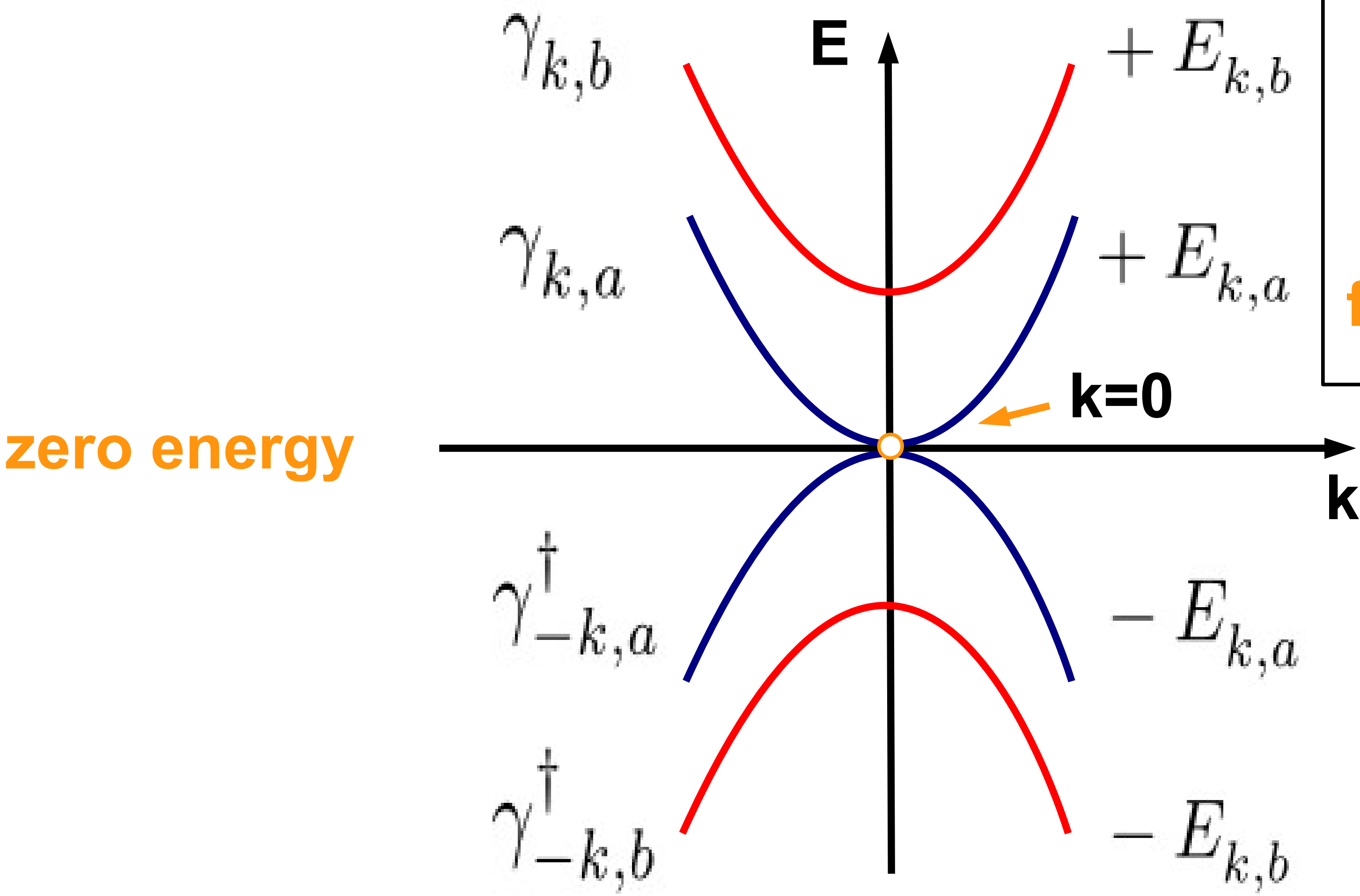
Majorana fermions in an infinite system

- **p-h** symmetry constraints the Bogoliubov eigenoperators and the energy spectrum



Majorana fermions in an infinite system

- If across the gap closing a **topological phase transition** occurs, a zero-energy Majorana fermion appears

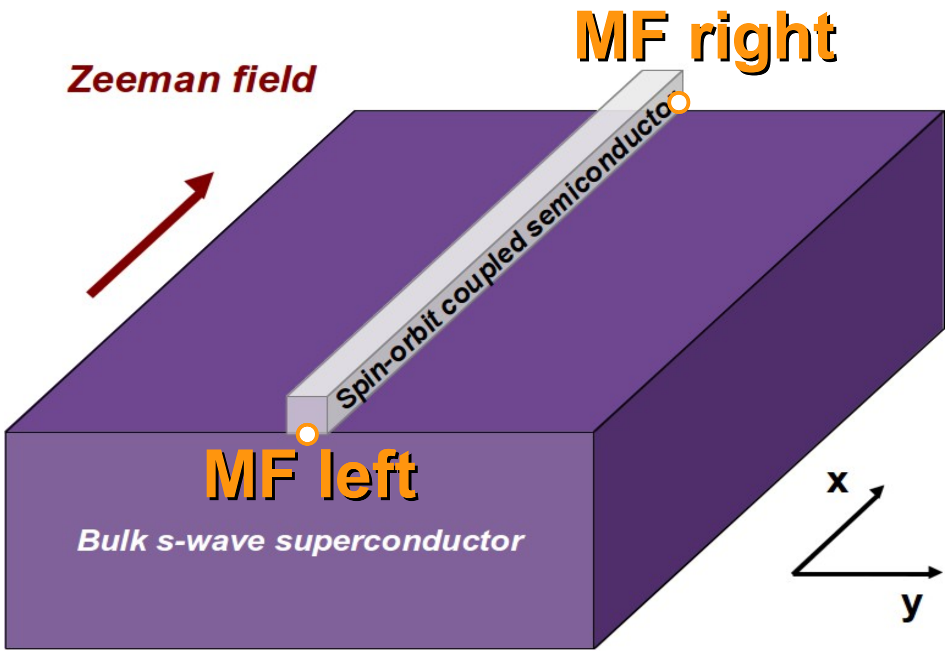


$$\gamma_{0,a} = \gamma^{\dagger}_{0,a}$$

Majorana fermion operator

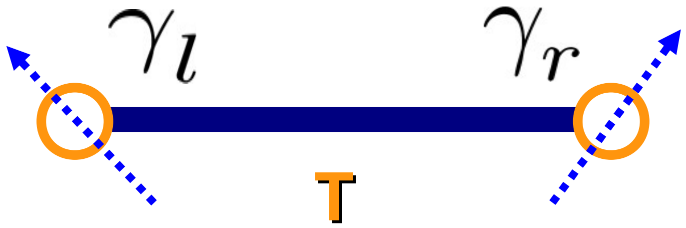
Majorana fermions in a bounded system

- If two systems of **different topological phases** neighbour, then Majorana fermions are located exactly at the interfaces

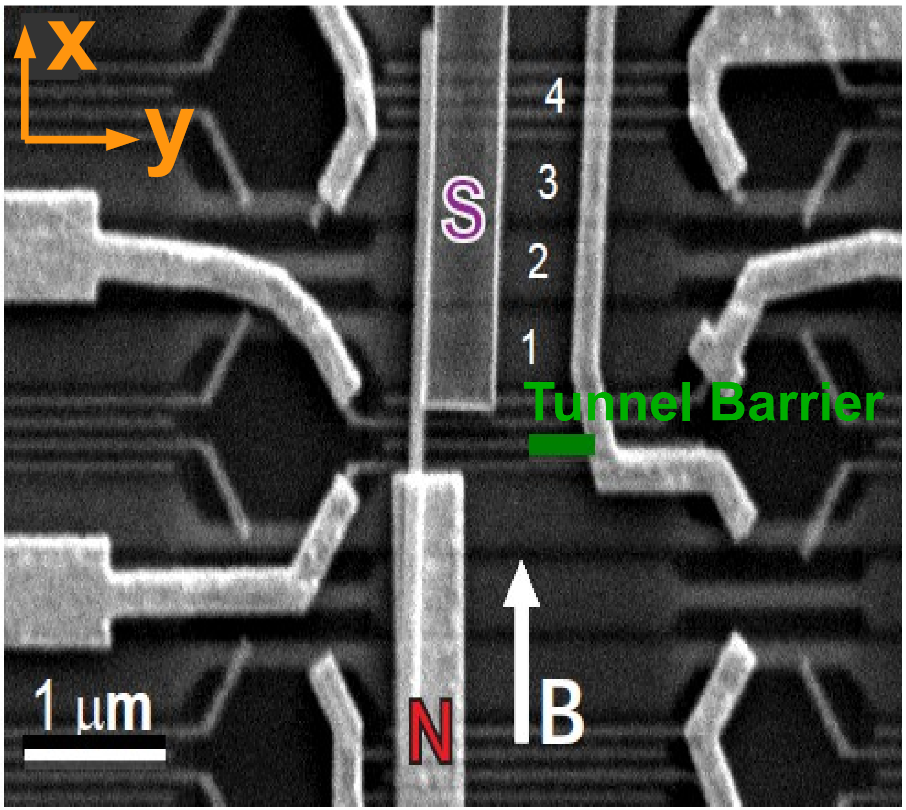


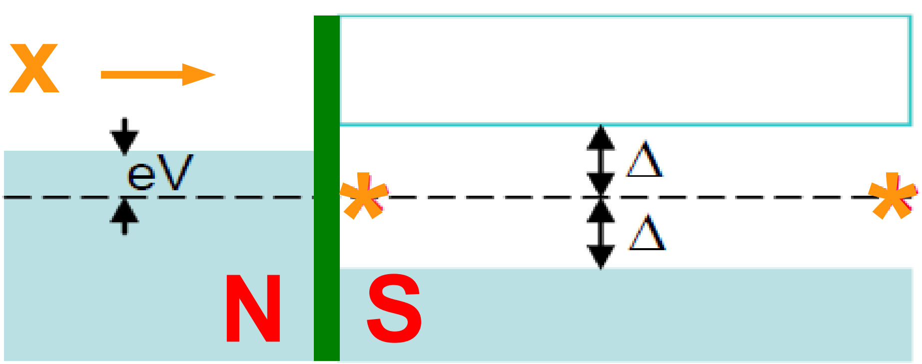
Surrounded by vacuum

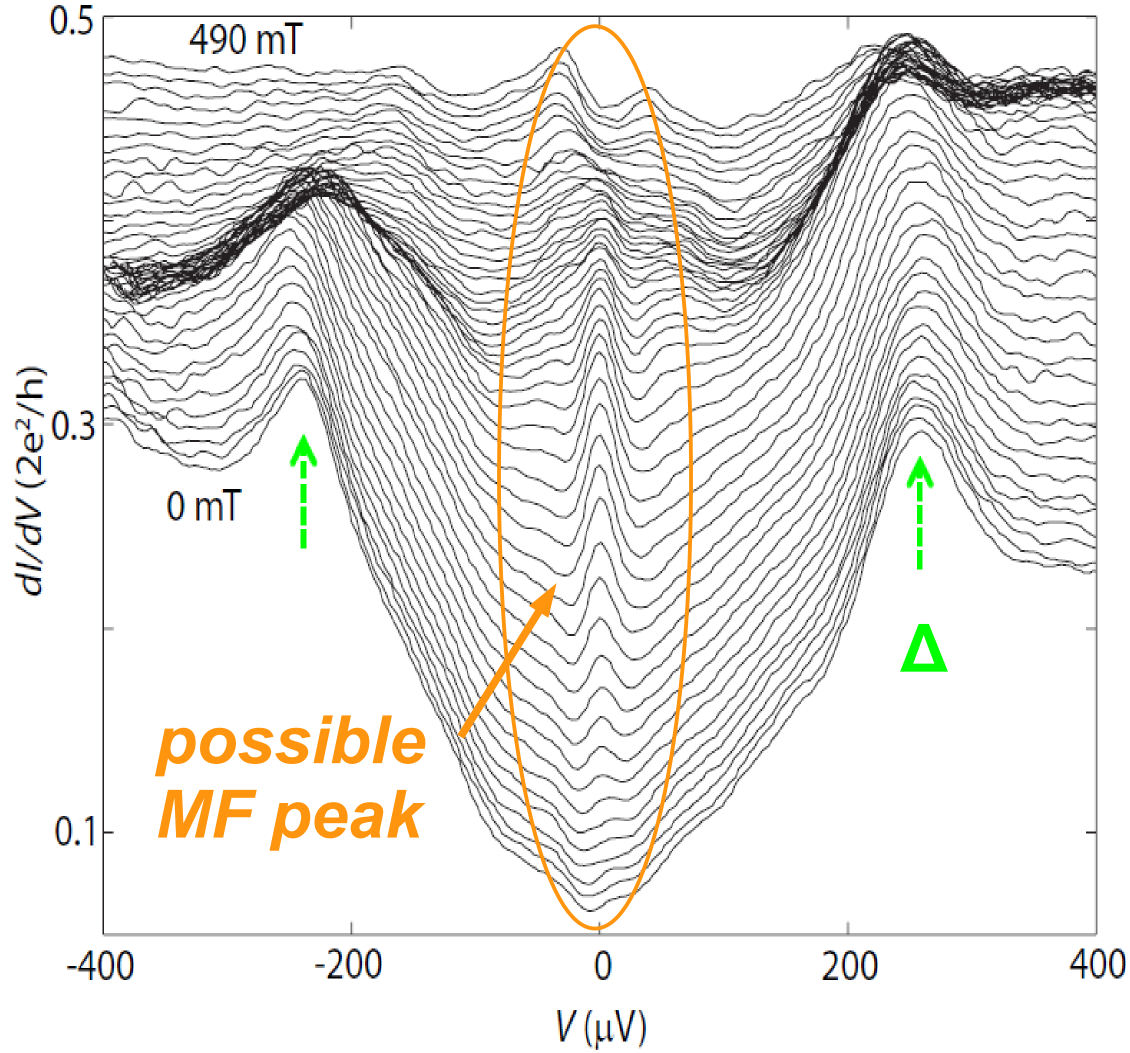
We shall graphically represent the above heterostructure with $\rightarrow$



Possible experimental signatures of MFs







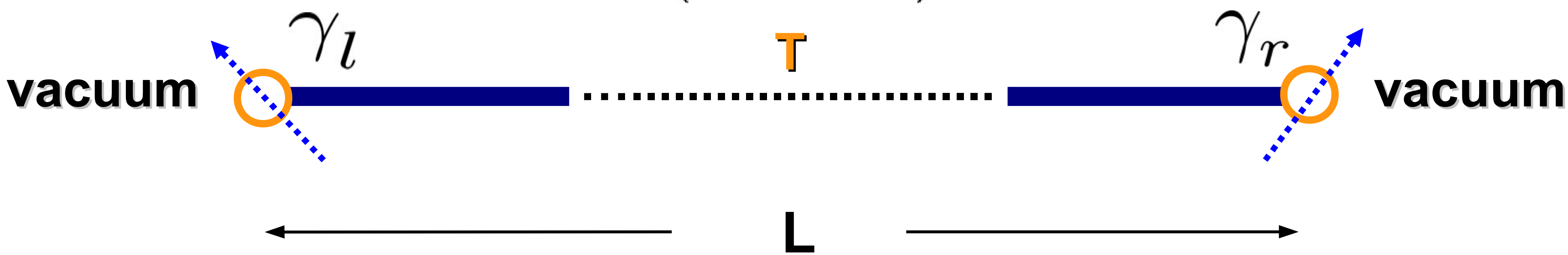
Mourik et al., Science (May 2012)

Next talks: M. Diez & D. Pikulin

Non-local zero-energy Dirac fermion and TQC

- If the topological superconductor has a length L, tending to ∞, the two Majorana fermions combine into a zero-energy Dirac fermion

$$d_0 = \left(\gamma_l + i\gamma_r\right)/\sqrt{2}$$



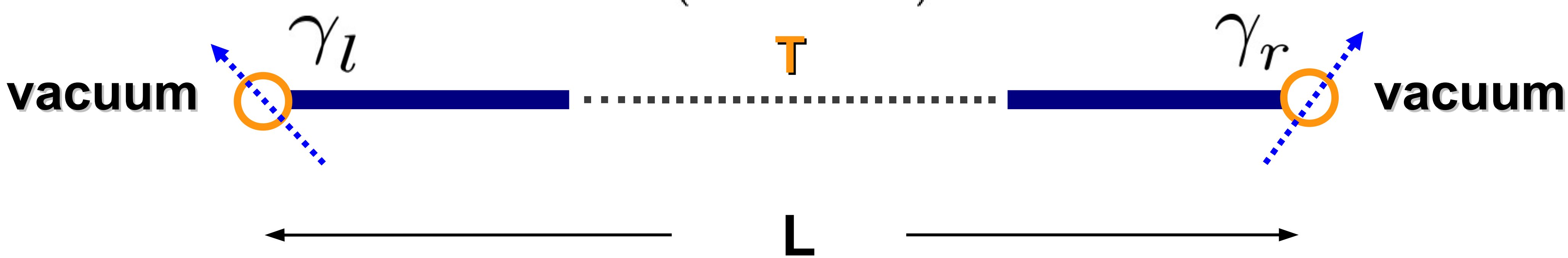
Doubly degenerate ground state $\rightarrow$ non-abelian topological order

Topological qubit: $\begin{pmatrix} |d_0;\text{empty}> \\ |d_0;\ \text{filled}> \end{pmatrix}$

Non-local zero-energy Dirac fermion and TQC

- If the topological superconductor has a length **L**, tending to **∞**, the two Majorana fermions combine into a zero-energy Dirac fermion

$$d_0 = \left(\gamma_l + i\gamma_r\right)/\sqrt{2}$$



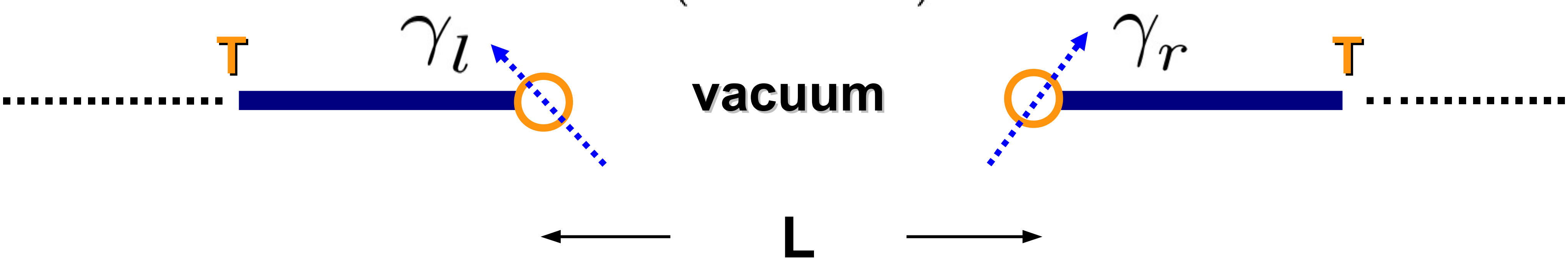
Doubly degenerate ground state $\rightarrow$ non-abelian topological order

Topological qubit: $\begin{pmatrix} |d_0; \text{empty} > \\ |d_0; \ \text{filled} > \end{pmatrix}$

Non-local zero-energy Dirac fermion and TQC

- If the topological superconductor has a length **L**, tending to **∞**, the two Majorana fermions combine into a zero-energy Dirac fermion

$$d_0 = \left(\gamma_l + i\gamma_r\right)/\sqrt{2}$$



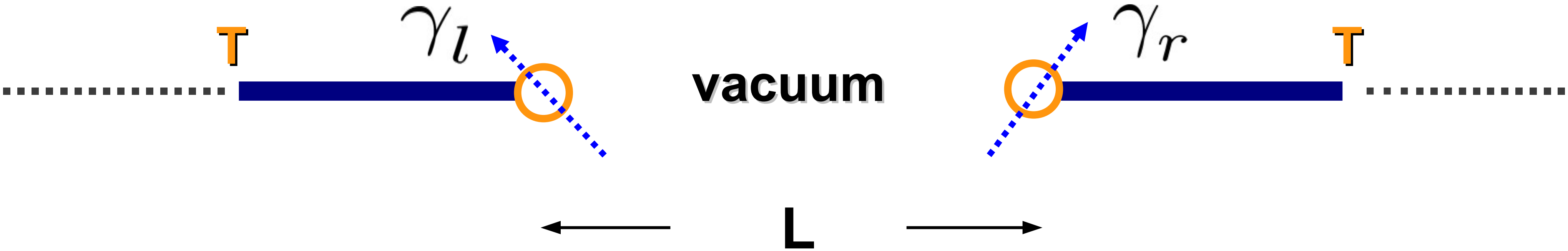
Doubly degenerate ground state $\rightarrow$ non-abelian topological order

Topological qubit: $\begin{pmatrix} |d_0; \text{empty} > \\ |d_0; \text{ filled} > \end{pmatrix}$

Non-local zero-energy Dirac fermion and TQC

- If the topological superconductor has a length L, tending to ∞, the two Majorana fermions combine into a zero-energy Dirac fermion

$$d_0 = \left(\gamma_l + i\gamma_r\right)/\sqrt{2}$$



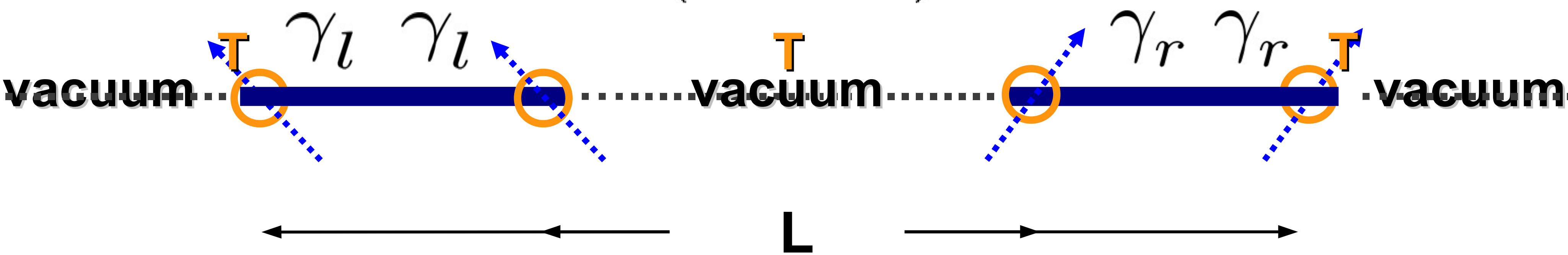
Doubly degenerate ground state $\rightarrow$ non-abelian topological order

Topological qubit: $\begin{pmatrix} |d_0; \text{empty}> \\ |d_0; \text{ filled}> \end{pmatrix}$

Non-local zero-energy Dirac fermion and TQC

- If the two topological superconductors are in distance L, tending to ∞, the two Majorana fermions combine into a zero-energy Dirac fermion

$$d_0 = \left(\gamma_l + i\gamma_r\right)/\sqrt{2}$$



Doubly degenerate ground state $\rightarrow$ non-abelian topological order

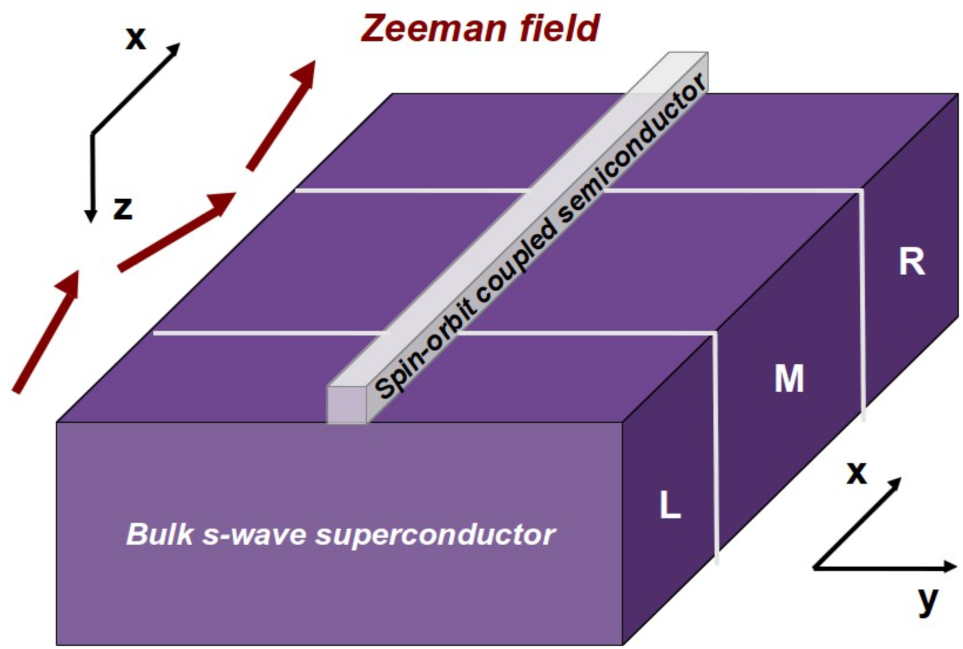
Topological qubit: $\begin{pmatrix} |d_0; \text{empty}> \\ |d_0; \text{filled}> \end{pmatrix}$

Our model for a heterostructure

- We set the **chemical potential equal to zero** and linearize about **k=0**. The more general **x**-dependent Hamiltonian reads

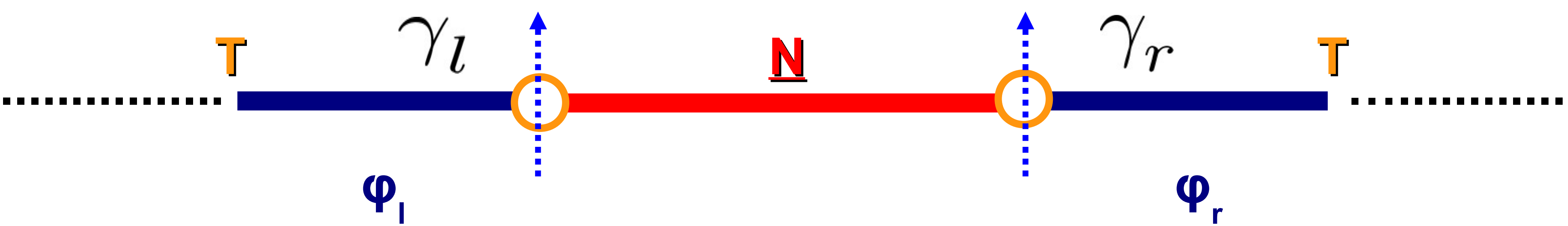
$$\mathcal{H} = \frac{1}{2}\int dx\, \widehat{\Psi}^\dagger(x)\left[v\hat{p}\tau_z\sigma_y - \frac{g\mu_B}{2}|\boldsymbol{\mathcal{B}}(x)|e^{-i\vartheta(x)\sigma_y}\tau_z\sigma_x - |\boldsymbol{\Delta}(x)|e^{-i\varphi(x)\tau_z}\tau_y\sigma_y\right]\widehat{\Psi}(x)$$

- Crucial ingredient: the Zeeman field rotates in the *x*-*z* plane!
- All the variables change stepwise
- The quantity $\frac{g\mu_B}{2}|\boldsymbol{\mathcal{B}}(x)| - |\boldsymbol{\Delta}(x)|$ alternates along the junction leading to **TNT** or **NTN** junctions
- We apply matching conditions for finite energy bound states



Interacting MFs (prior results I, Kitaev limit)

- **TNT** junction
- **T** segment: spin-orbit coupling + Zeeman field + superconductivity
- **N** segment: spin-orbit coupling + Zeeman field (***no supercond.***)
- Kitaev spinless model: **spins frozen**



- The previously zero-energy Dirac fermion acquires finite energy:

$$E = J_M \cos\left(\frac{\varphi_l - \varphi_r}{2}\right) \qquad \text{with} \qquad J_M = |\Delta| e^{-|\Delta| L / v\hbar}$$

Kitaev, Phys. Usp. (2001), Liang Jiang et al., PRL (2011)

MF Josephson effect (prior results I)

- The low-energy many-body Hamiltonian $\mathcal{H}_{MF} = E\left(d_0^\dagger d_0 - \frac{1}{2}\right)$

- The Majorana fermion mediated Josephson current reads

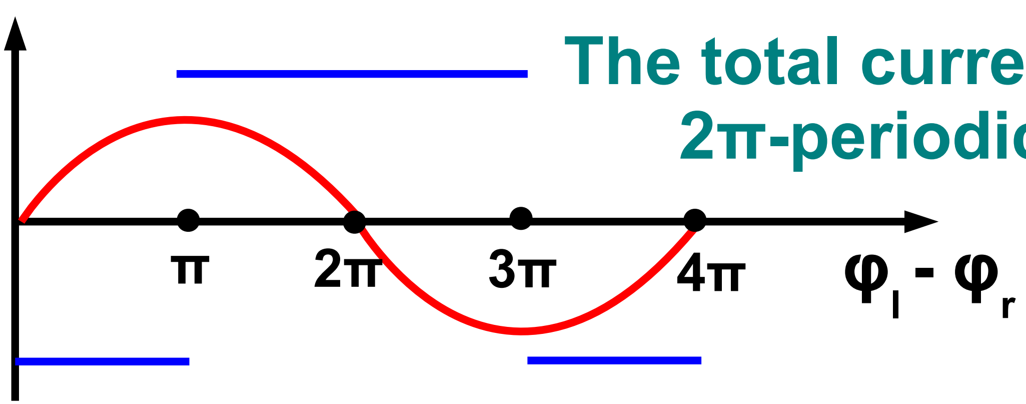
$$J_c = \frac{2e}{\hbar}\left\langle \frac{\partial \mathcal{H}_{MF}}{\partial(\varphi_l - \varphi_r)} \right\rangle = \frac{e}{\hbar} J_M \sin\left(\frac{\varphi_l - \varphi_r}{2}\right)\left(< d_0^\dagger d_0 > - \frac{1}{2}\right)$$

$$< d_0^\dagger d_0 > - \frac{1}{2} = \Theta\left[J_M \cos\left(\frac{\varphi_l - \varphi_r}{2}\right)\right] - \frac{1}{2}$$

4π-periodic, change sign for a 2π-shift

4π-periodicity appears if fermion parity is fixed

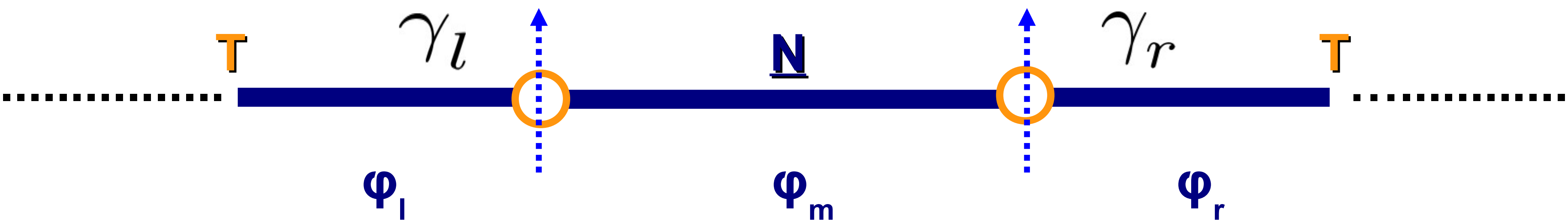
San-Jose et al., PRL (2012)
Pikulin and Nazarov, PRB (2012)



The total current is 2π-periodic

Interacting MFs (prior results II, Kitaev limit)

- **TNT** junction
- **T** segment: spin-orbit coupling + Zeeman field + superconductivity
- **N** segment: spin-orbit coupling + Zeeman field + superconductivity
- Kitaev spinless model: **spins frozen**



- The previously zero-energy Dirac fermion acquires finite energy:

$$E = J_M \cos\left(\frac{\varphi_l - \varphi_r}{2}\right) + J_Z \cos\left(\frac{\varphi_l + \varphi_r}{2} - \varphi_m\right)$$ with

$$J_Z = (|\mathcal{B}|L/v\hbar) J_M$$

Liang Jiang et al., PRL (2011)

MF Josephson effect (prior results II)

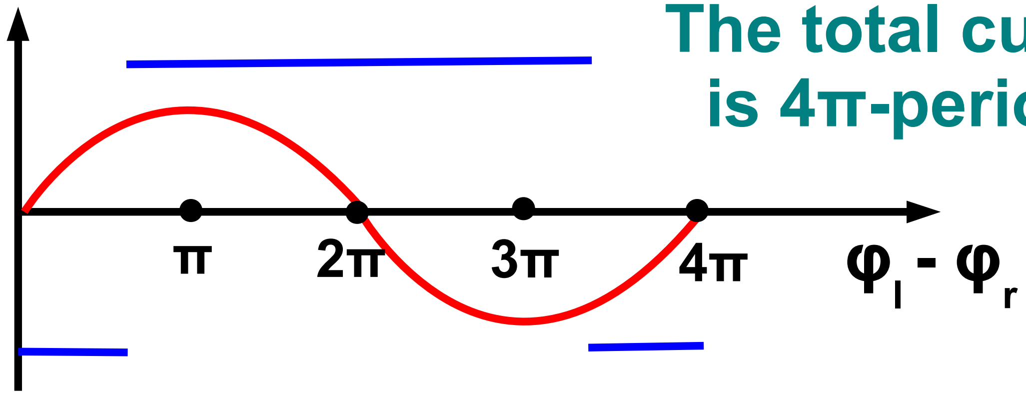
- For a balanced junction $\varphi_m=(\varphi_l+\varphi_r)/2$: $E = J_M \cos\left(\frac{\varphi_l-\varphi_r}{2}\right) + J_Z$
- The Majorana fermion mediated Josephson current reads

$$J_c = \frac{2e}{\hbar}\left\langle \frac{\partial \mathcal{H}_{MF}}{\partial(\varphi_l-\varphi_r)} \right\rangle = \frac{e}{\hbar} J_M \sin\left(\frac{\varphi_l-\varphi_r}{2}\right)\left(< d_0^\dagger d_0 > -\frac{1}{2}\right)$$

$$< d_0^\dagger d_0 > -\frac{1}{2} = \Theta\left[J_M \cos\left(\frac{\varphi_l-\varphi_r}{2}\right) + J_Z\right] - \frac{1}{2}$$

4π-periodic

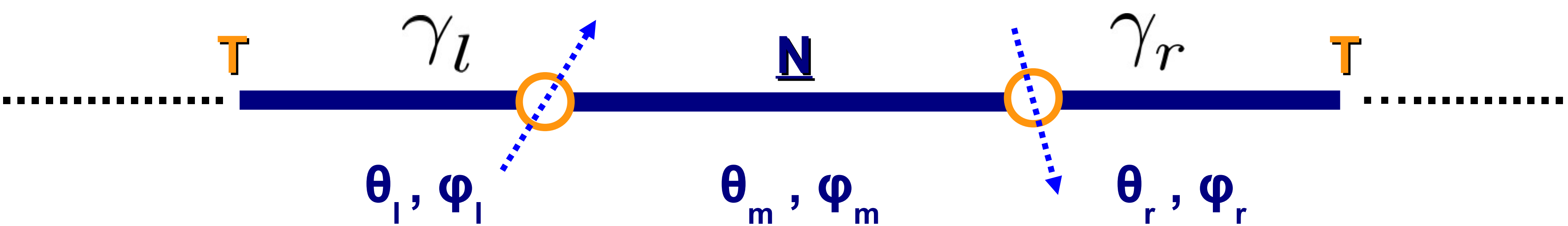
The fermion parity does not change sign under a 2π-shift



The total current is 4π-periodic

Interacting MFs (our results, no Kitaev limit)

- **TNT** junction
- Spinful model: **Zeeman field angle varies along the junction**



- We recover the previous relations **plus** an additional prefactor:

$$E = \qquad \ll 1$$

$$\times \left[J_M \cos\left(\frac{\varphi_l - \varphi_r}{2}\right) + J_Z \cos\left(\frac{\varphi_l + \varphi_r}{2} - \varphi_m\right)\right]$$

Exact treatment reveals magnetic coupling !!!

Interacting MFs (our results, balanced case)

- For a "balanced" junction, $\varphi_m=(\varphi_l+\varphi_r)/2$ and $\theta_m=(\theta_l+\theta_r)/2$ we get

$$E = \left[1 - \frac{|\mathcal{B}|}{|\Delta|} \frac{\cos\left(\frac{\varphi_l-\varphi_r}{2}\right) + \cos\left(\frac{\vartheta_l-\vartheta_r}{2}\right)}{2}\right] \left[J_M \cos\left(\frac{\varphi_l-\varphi_r}{2}\right) + J_Z\right]$$

- We obtain a magnetic contribution to the Josephson current

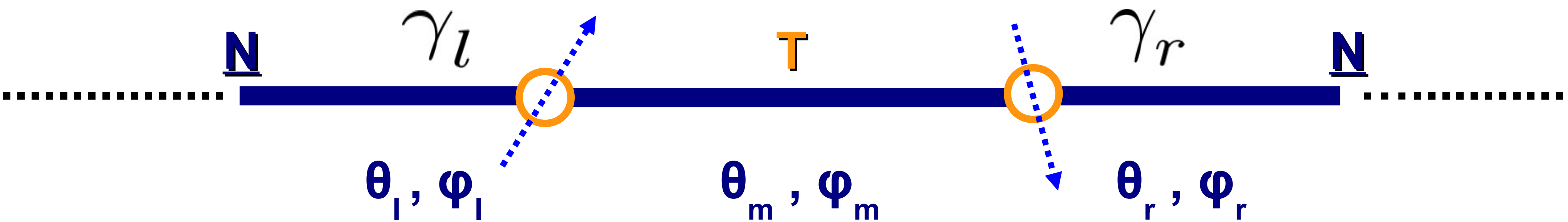
$$J_{cm} \sim J_M \cos\left(\frac{\vartheta_l-\vartheta_r}{2}\right) \sin\left(\frac{\varphi_l-\varphi_r}{2}\right)$$

- We obtain a spin current running along x-axis and polarized along the y-axis, controlled by the variation of the superconducting phase

$$J_x^y \sim \left[J_M \cos\left(\frac{\varphi_l-\varphi_r}{2}\right) + J_Z\right] \sin\left(\frac{\vartheta_l-\vartheta_r}{2}\right) \left(< d_0^\dagger d_0 > -\frac{1}{2}\right)$$

Interacting MFs (our results, no Kitaev limit)

- **NTN** junction (only recently studied)
- Apart from us also: *Q. Meng et al. (2012)*, *L. Jiang et al. (2012)*



- The energy splitting of the zero-energy Dirac fermion reads:

$$E =$$

<<1

$$\times \left[J_M \cos\left(\frac{\vartheta_l - \vartheta_r}{2}\right) + J_Z \cos\left(\frac{\vartheta_l + \vartheta_r}{2} - \vartheta_m\right)\right]$$

in the Kitaev limit

Conclusions

- We have studied the Majorana fermion mediated transport, by allowing the magnetic field orientation to vary

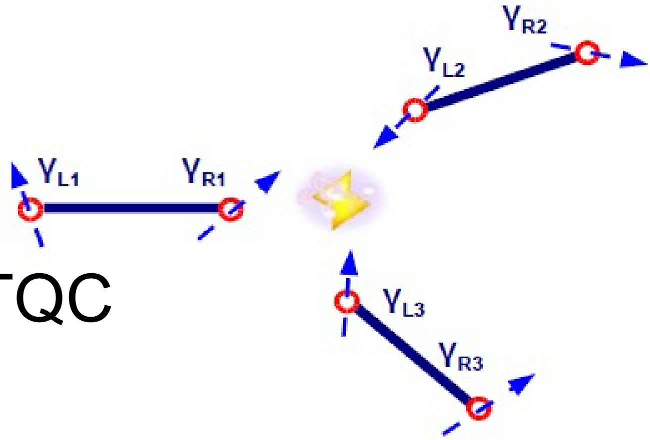
- In the simplest case, the interaction between 2 Majoranas reads

$$J_{\varphi\vartheta} \cos\left(\frac{\varphi_r - \varphi_l}{2}\right) \cos\left(\frac{\vartheta_r - \vartheta_l}{2}\right)$$

In agreement with results for p-wave superconductors,
Kwon et al., EPJ B (2004)

- Emergent magnetically tuned Majorana Josephson effect may constitute a unique fingerprint of MFs

- The magnetic field phase dependence of the MF coupling opens up alternative routes for TQC



Special thanks to Jens Michelsen

and of course to you...

P. Kotetes, A. Shnirman and G. Schön, arXiv:1207.2691