CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Institut de Recherche sur les Phénomènes Hors Équilibre

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Some pattern forming mechanisms on interfaces

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Interface response

- to external forcing (energy flux, heating/cooling, mechanical stress ...)
- by form adaptation / instability development
- yielding pattern formation

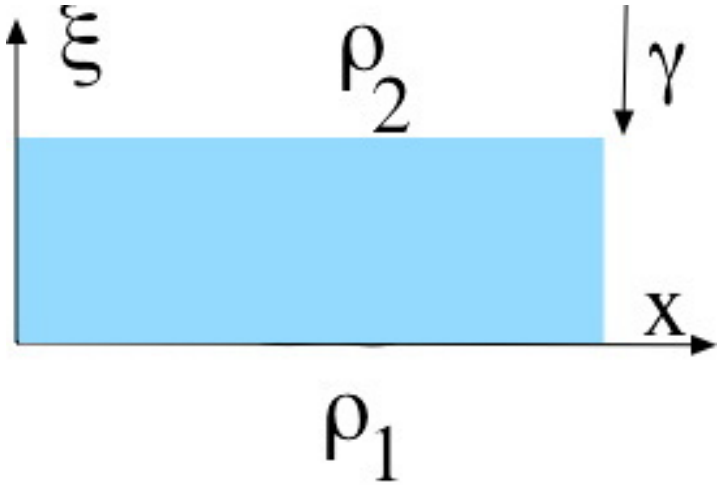
Interfaces between :
- liquid/liquid : hydrodynamics
- liquid/solid : phase transition
- solid/solid : elasticity

Universal mechanisms : possibly partially relevant to interface dynamics following laser impact ?

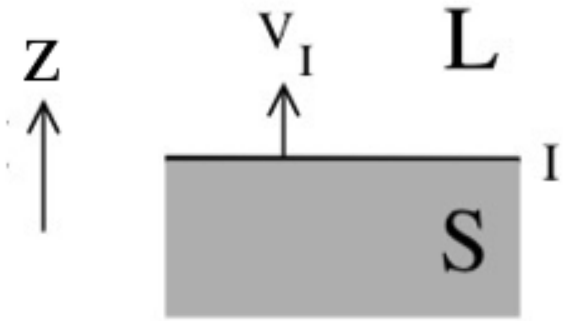
possibly coupled one to the other

Time scale / length scale

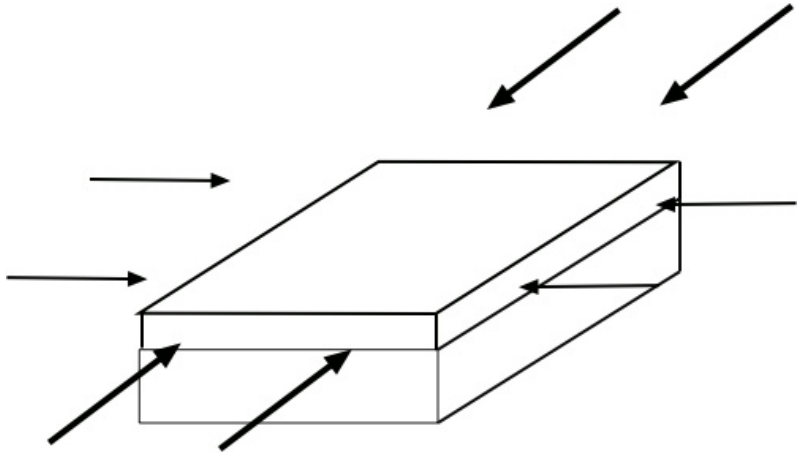
liquid / liquid



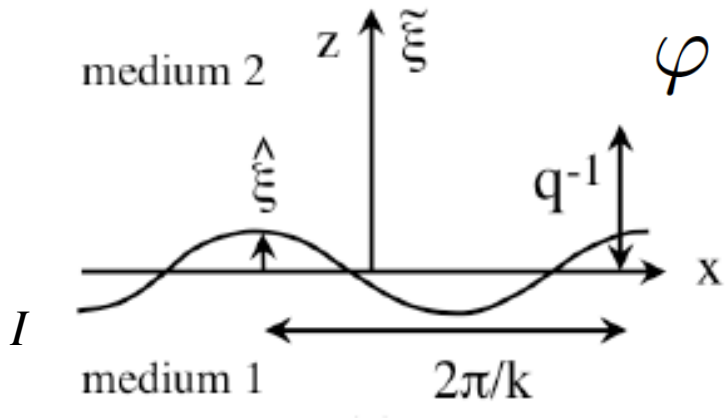
liquid / solid



solid / solid



Linear instability of dynamical interfaces



Dynamical system

Bulk dynamics	Navier-Stokes eqs Advection/diffusion eqs Föppl-vonKarman eqs	: media 1 and 2 field φ
Jump relations :	couplings between media 1 and 2 normal interface velocity V_I	: at the interface
Interface localization	phase change	: at the interface

$$\tilde{\varphi} = \hat{\varphi}\exp(\sigma t - qz + ikx)$$

$$\tilde{\xi} = \hat{\xi}\exp(\sigma t + ikx)$$

eigen modes of perturbation

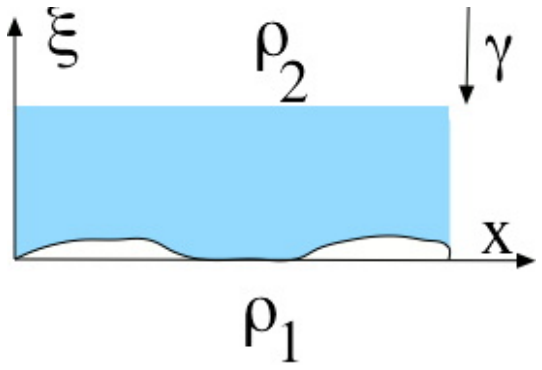
$\sigma(k,q)$

$q(k)$

$\sigma(k)$

L/L interface

Rayleigh-Taylor instability



$$\xi \sim e^{kx+\sigma t}$$

$$\frac{D\mathbf{v}}{Dt} = -\frac{\nabla p}{\rho} + \nu \triangle \mathbf{v} + \gamma$$

$$\nabla . \mathbf{v} = \mathbf{0}$$

$$\nabla \wedge \mathbf{v} = \mathbf{0}$$

$$\delta p = 0$$

$$\sigma \sim (A\gamma k + \nu^2 k^4)^{1/2} - \nu k^2$$

$$A = \frac{\rho_2 - \rho 1}{\rho_2 + \rho_1}$$

$$\gamma \sim 10 m/s^{-2}$$

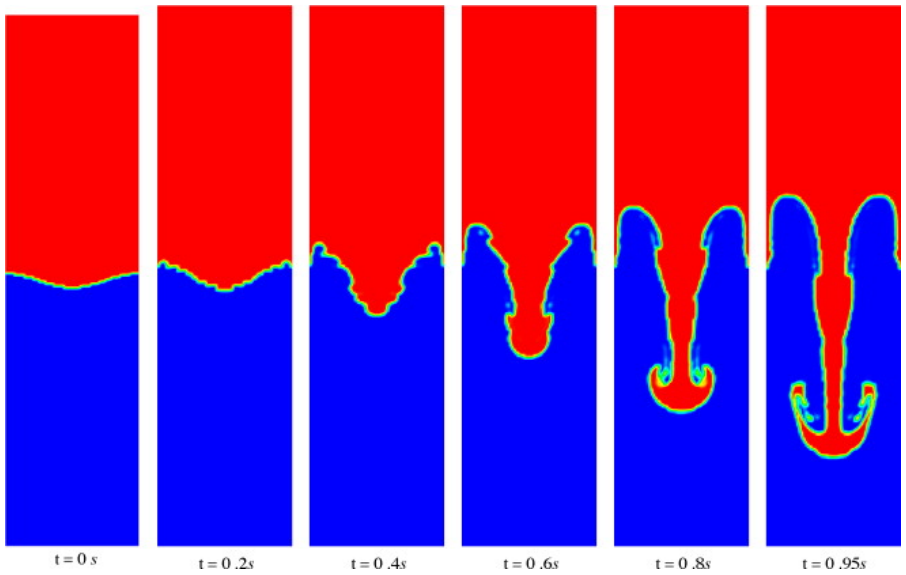
$$k_M \sim (\frac{A\gamma}{\nu^2})^{1/3}$$

$$\nu \sim 1 mm^2/s$$

$$\sigma_M \sim \frac{(A\gamma)^{2/3}}{\nu^{1/3}}$$

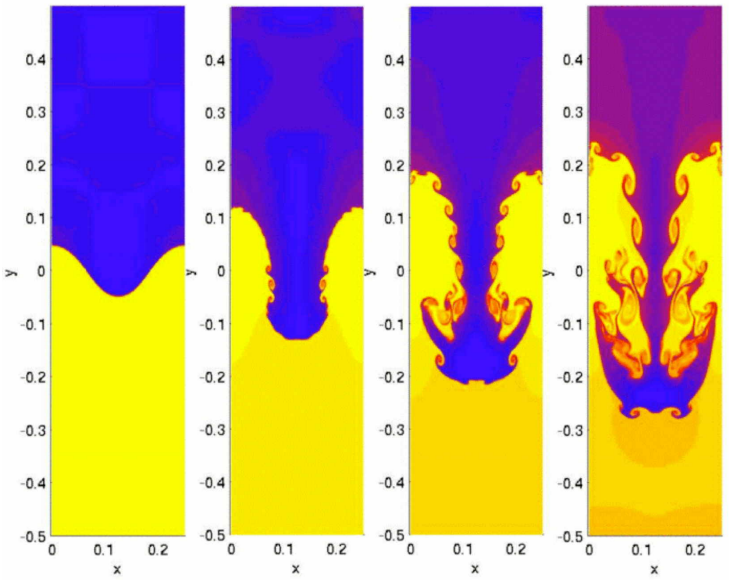
$$\lambda_M = 2\pi/k_M \sim 0.3mm$$

$$\tau_M = \sigma_M^{-1} \sim 2ms$$



R.Gopala, B. van Machem, Gothenburg, Sweden

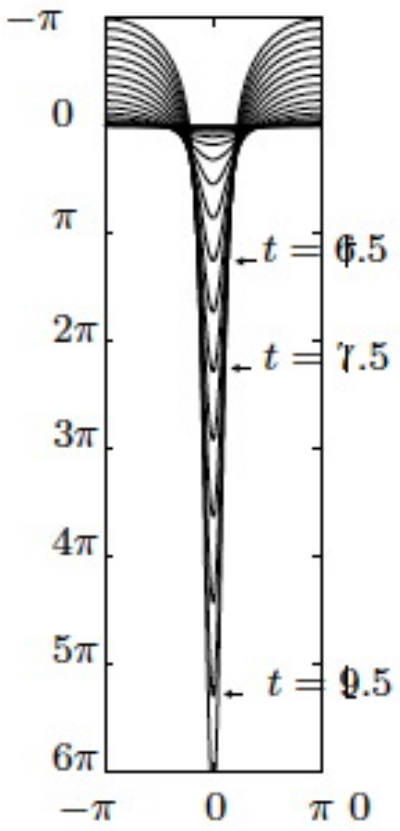
Formation of blobs



Li, Shengtai and Hui Li. Los Alamos National Laboratory.

Additional Kelvin-Helmholtz instability

instability couplings

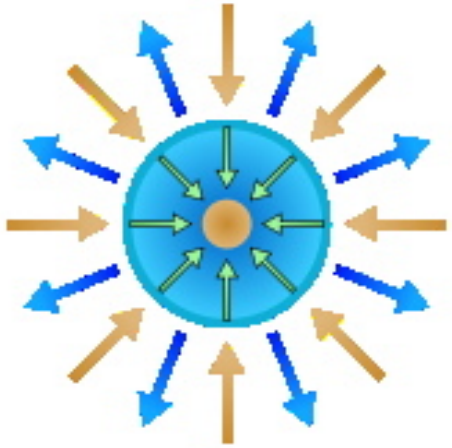


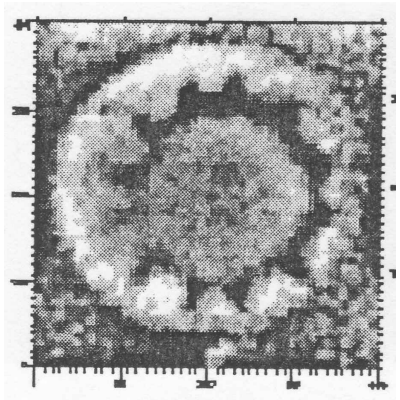
Perturbation analysis
+
similarity

curvature $\kappa \approx t^{3}$

Duchemin et al.,Phys.Rev.Lett.94,224501,2005

Rayleigh-Taylor instability and ICF





Cylindrical target of D-T

Thermal gradient, $D(T) \approx T^n$

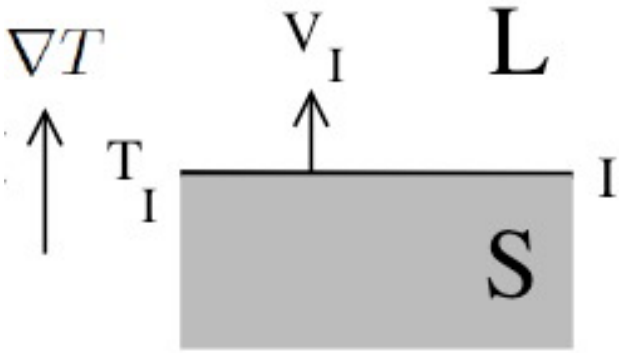
$$\sigma \sim \alpha(\gamma k)^{1/2} - \beta k V_a$$

Possibility of stabilization by increase of transverse heat diffusion

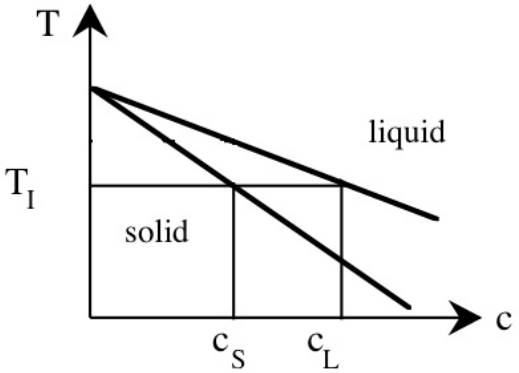
Layered target

L.Masse, Phys.Rev.Lett, 98,245001,2007

L/S interface



Mixture ≈ binary : c = [solute]



$$c_S = K c_L < c_L$$ → solute rejection

→ need to evacuate solute excess to proceed growth

→ process : diffusion

$$\Delta c_I = c_L - c_S$$

→ diffusion limited growth

Advection/diffusion

$$\frac{\partial c}{\partial t} - V\mathbf{z}.\nabla c = D\nabla^2 c$$

Gibbs-Thomson relation

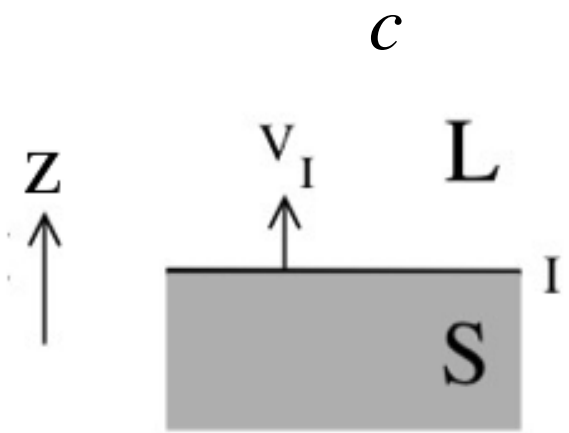
$$T_I = T_M - mc_I - T_M\frac{\gamma}{Q}(\kappa_1 + \kappa_2) \quad + \ldots$$

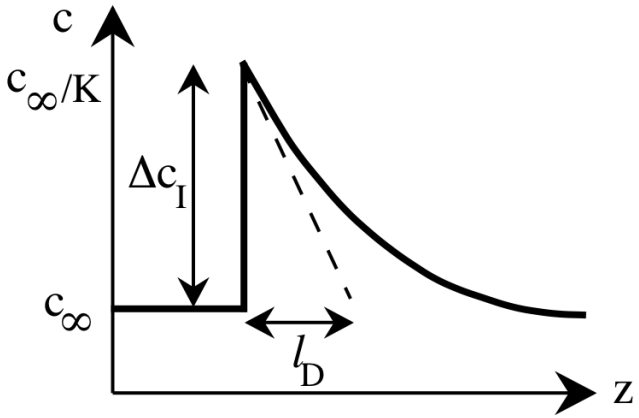
Mass conservation

$$V_I \; c_I \; (1-K) = -D \; \nabla c.\mathbf{n_I}$$

kinetic corrections

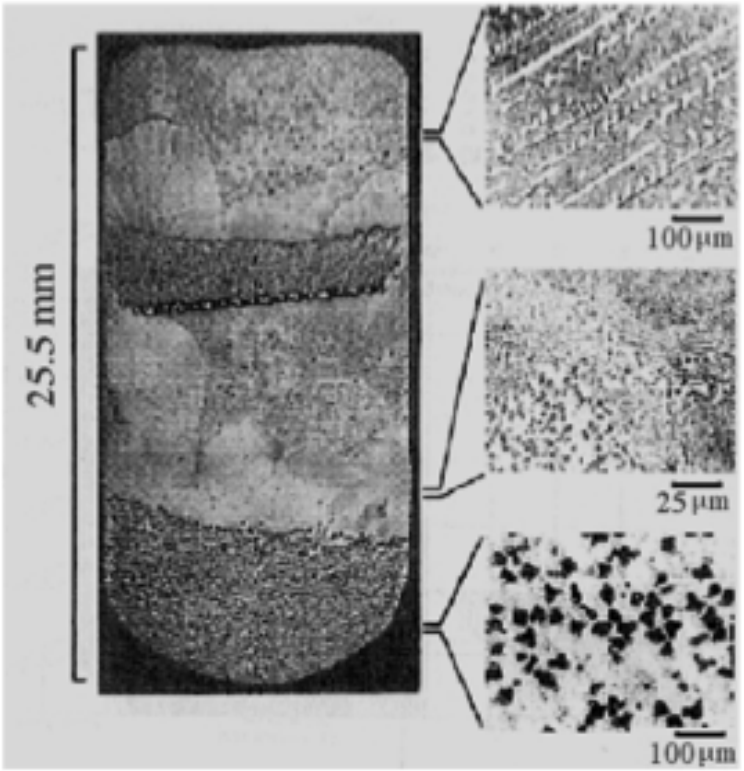
Solute layer

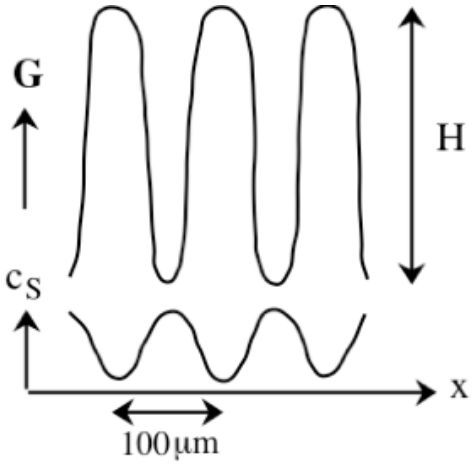


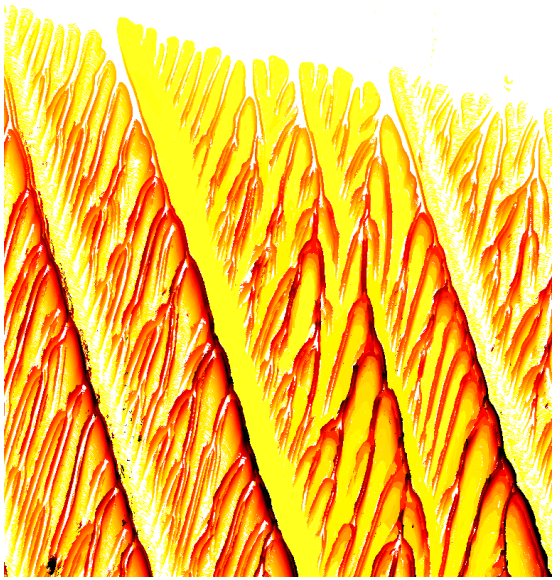


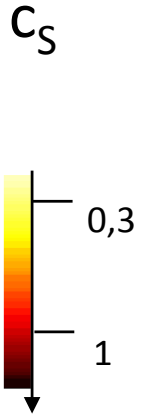
$l_D = D/V$

Micro-segregation

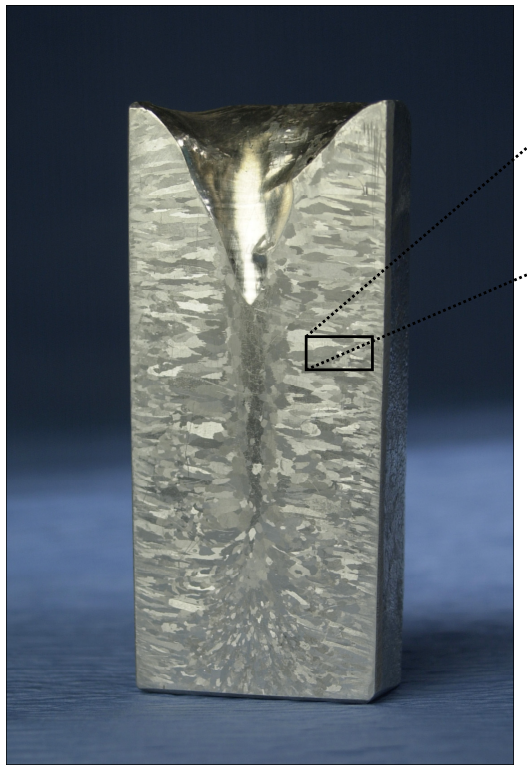




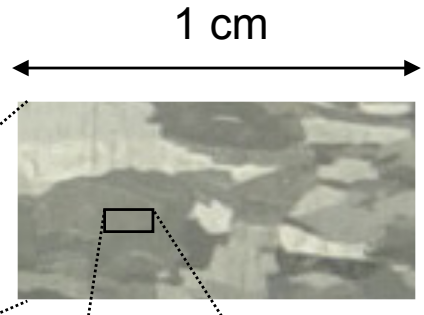


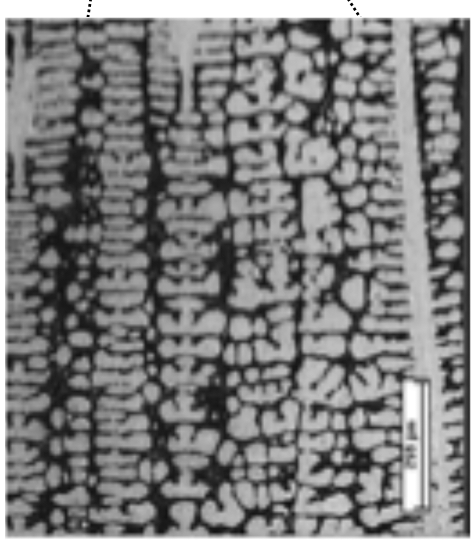


Heterogeneities and crystal orientation



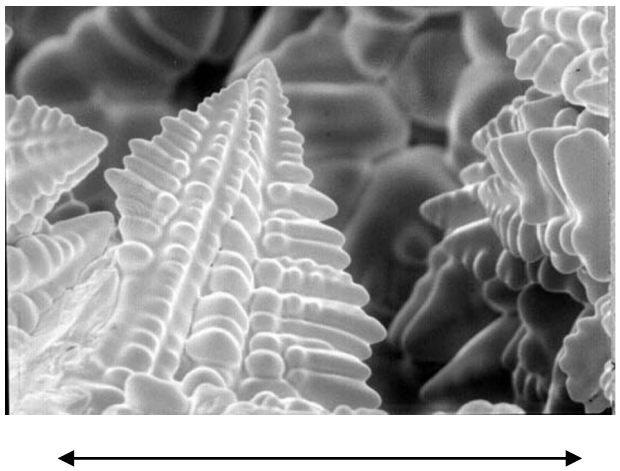
Aluminium ingot





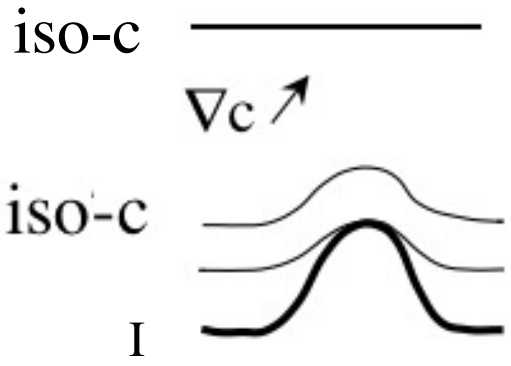
Grains : ≠ crystal orientation

dendrites



Planar (Mullins-Sekerka) instability

Mass conservation → $V_I \; c_I \; (1-K) = -D \; \nabla c.\mathbf{n_I}$ → $V_I \propto \nabla c.\mathbf{n_I}$



$\nabla c \nearrow \; V_I \nearrow$: instability

Stabilizing effects : thermal gradient ∇T
surface tension d_o

$$\sigma \sim k(V - V_c) - D d_o k^3 - V^2/D$$

$$k_M \sim \left(\frac{V - V_c}{D d_o}\right)^{1/3}$$

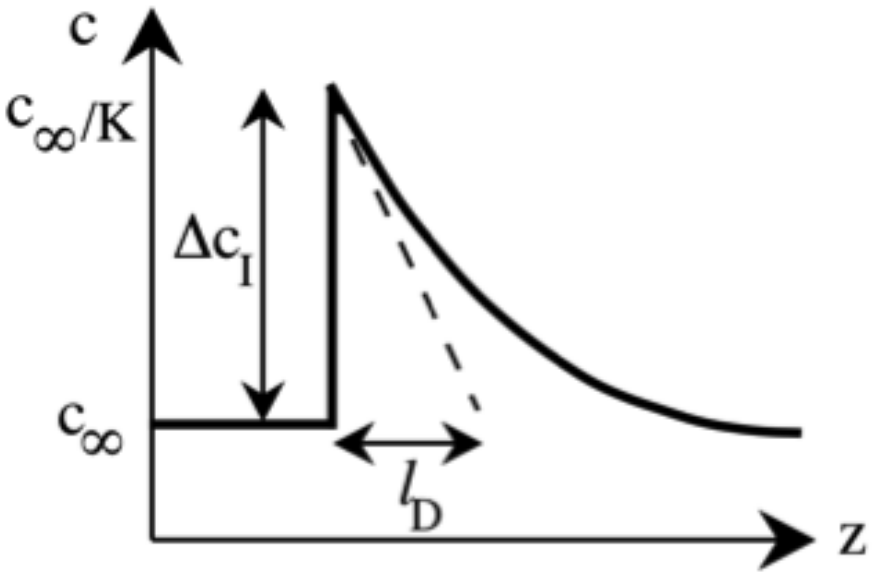
$$\sigma_M \sim \frac{(V - V_c)^{3/2}}{(D d_o)^{1/2}}$$

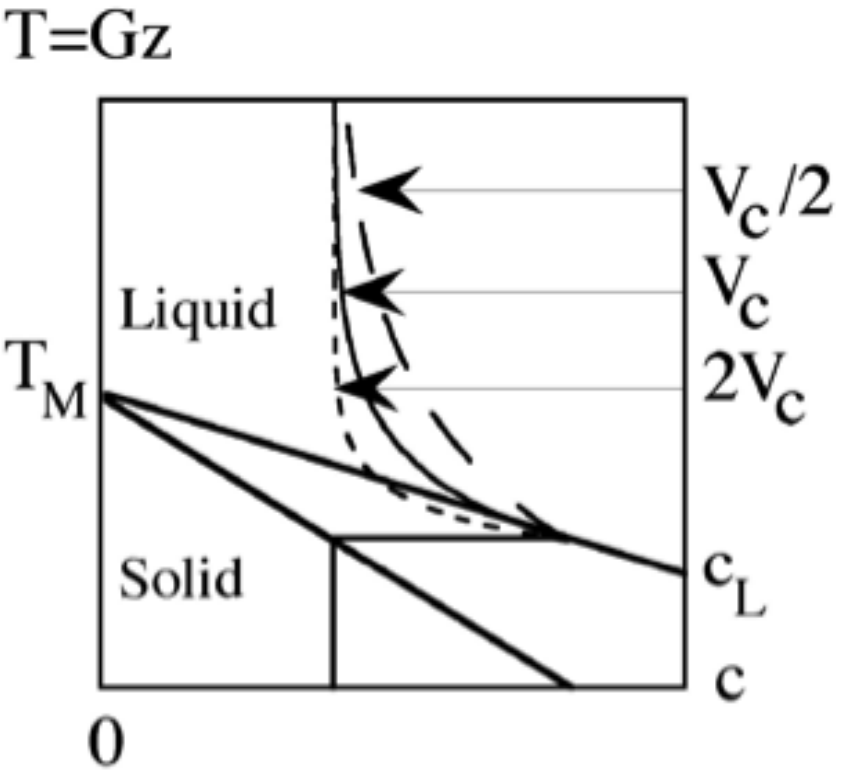
$D \sim 10^3 \mu m^2/s$
$d_0 \sim 10^{-2} \mu m$

$c \sim 1\%$ $\quad V_c \sim 1 \mu m/s$

$V \sim 100 \mu m/s$ →
$\lambda_M \sim 300 \mu m$
$\tau_M = \sigma_M^{-1} \sim 3ms$

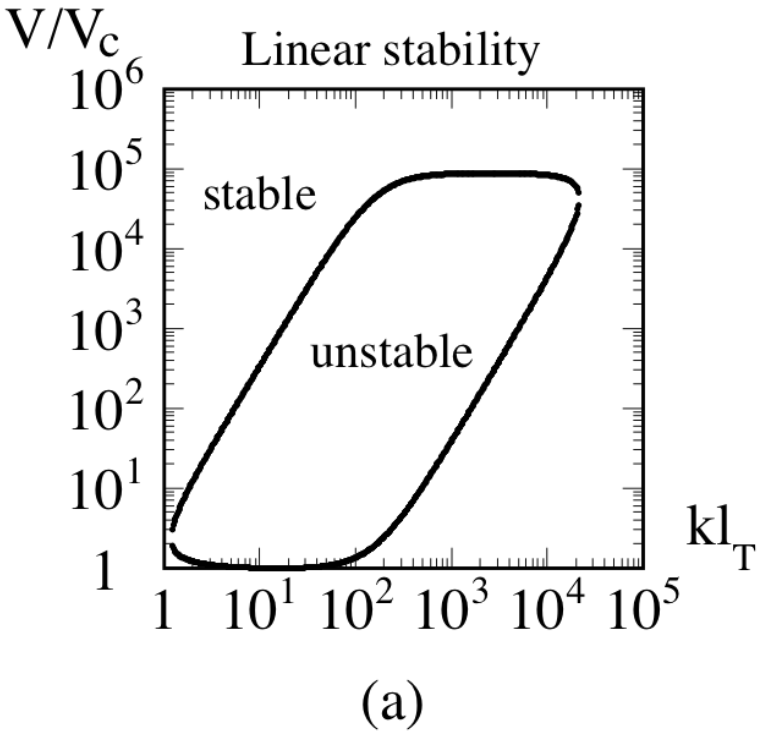
Constitutional supercooling criterium

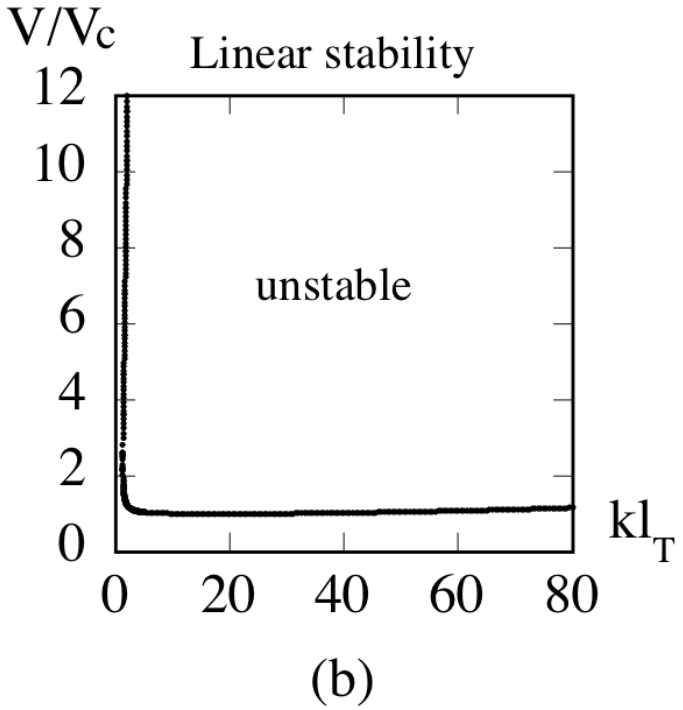




$l_D = D/V$

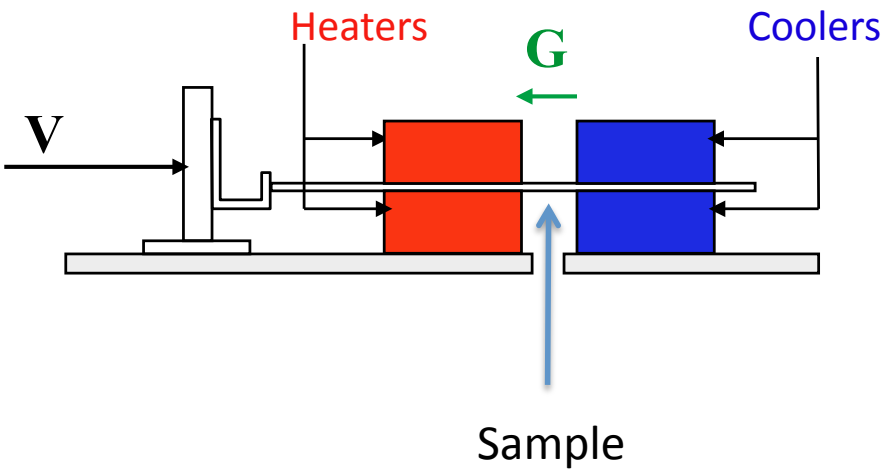
$l_T = D/|\nabla T| \sim mm$ $\quad$ $d_0 \sim 10\mathring{A}$ $\quad$ $d_0 \ll l_T$

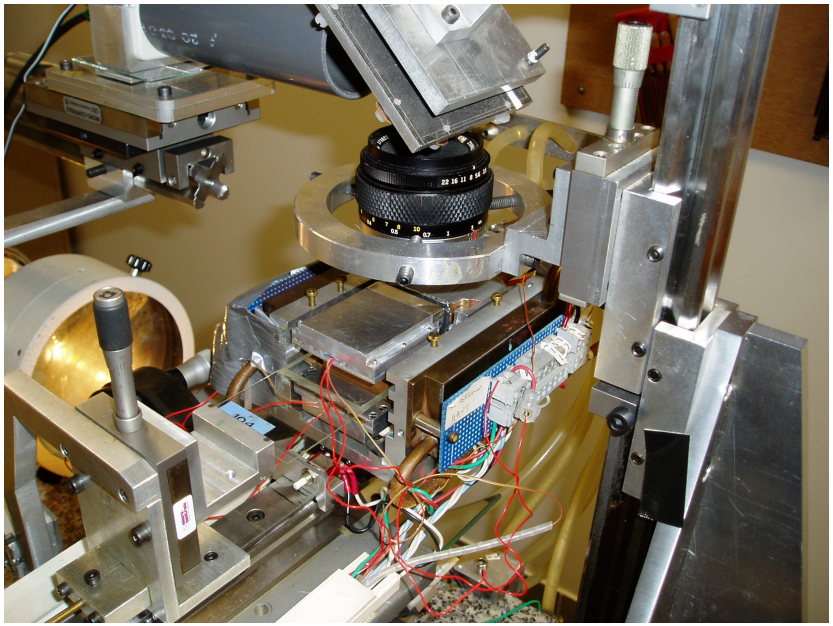


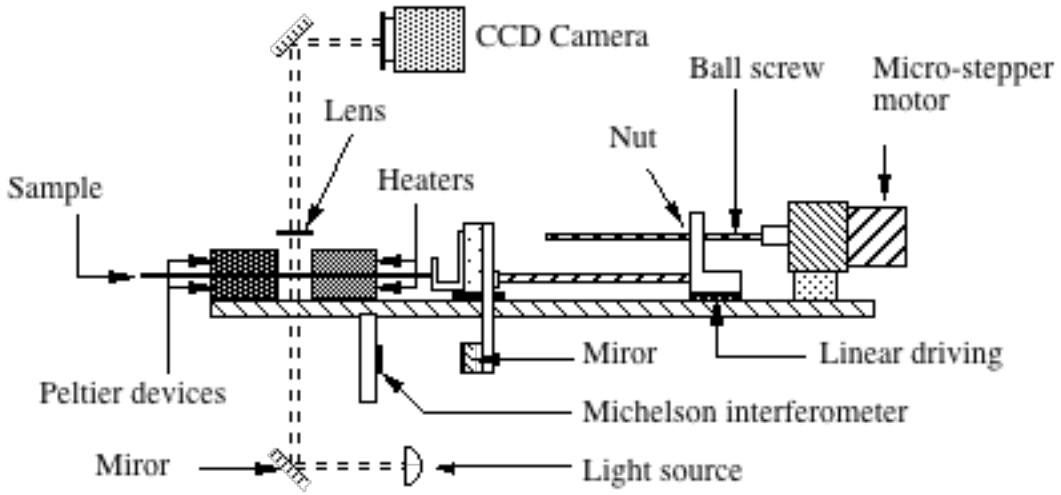


Restabilization at large velocities by surface tension
and diffusion : $\quad V \approx 10^5$ Vc $\approx$ 10 cm/s

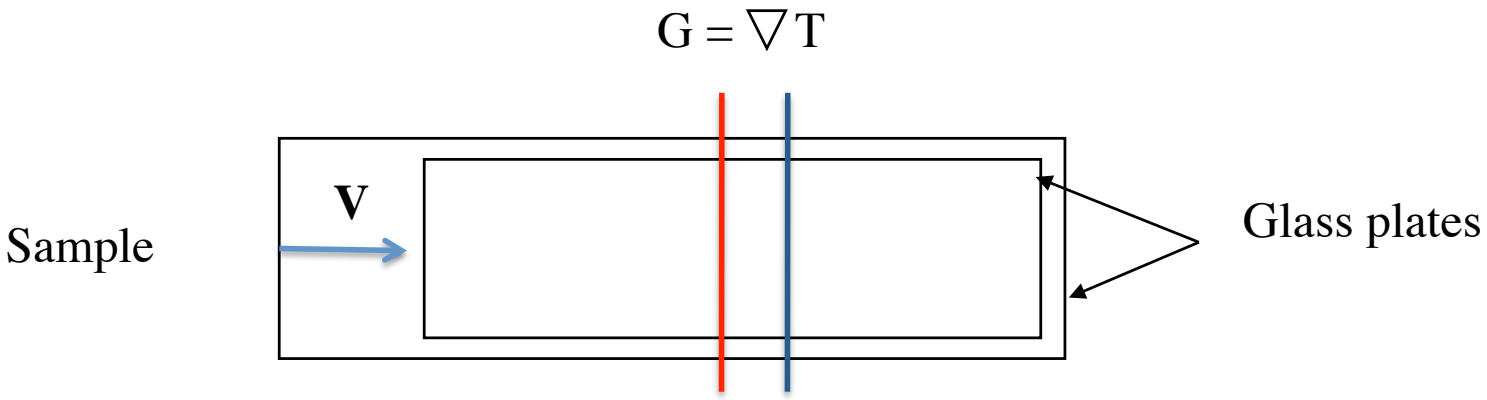
Directional solidification experiment







Sample and mixture



Thin sample

e ≈ x*10 microns

Single layer :

Transparent mixture : succinonitrile

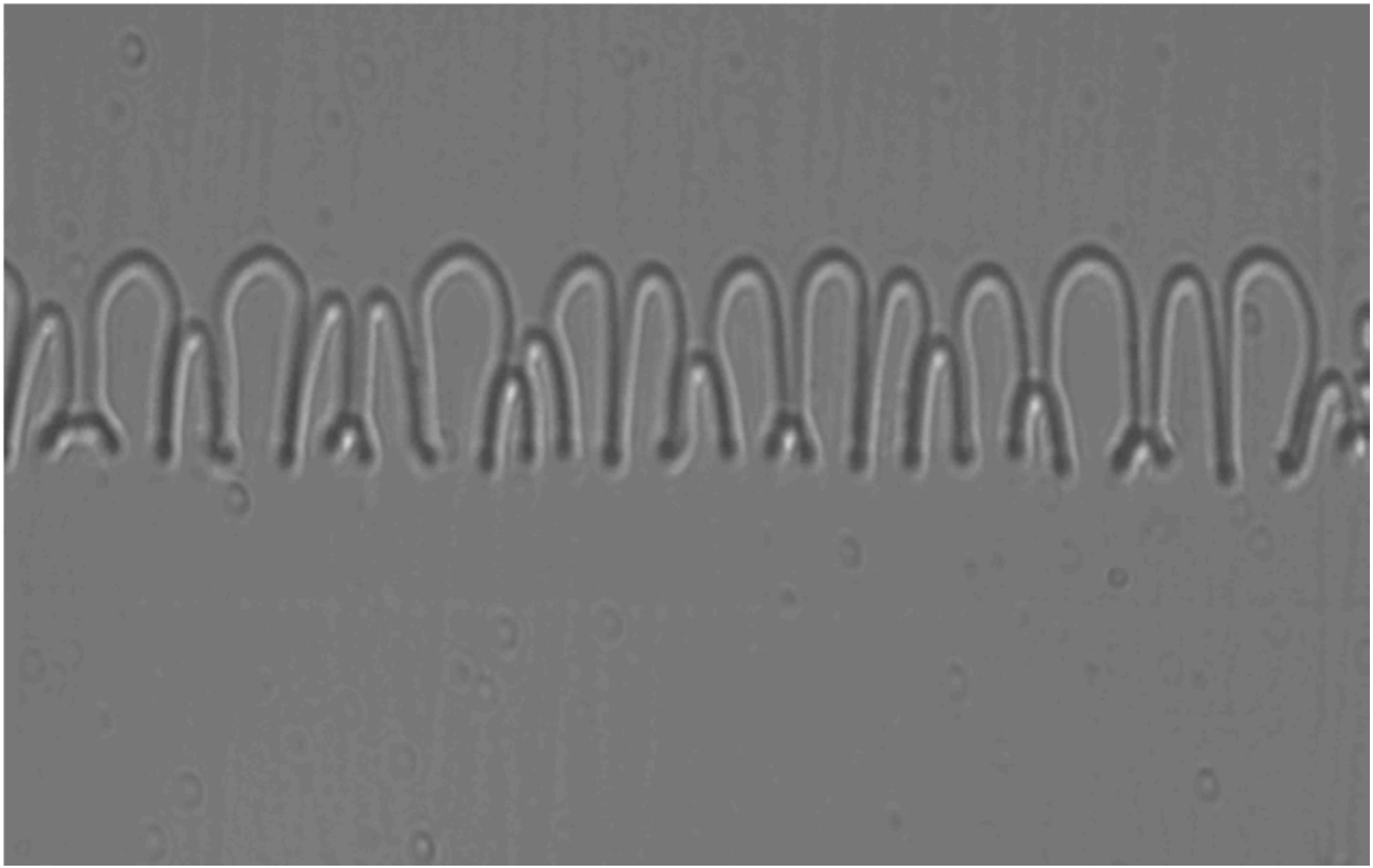
Interfacial properties close to those of metals

Pushing velocities : 0 to 50 μm/s

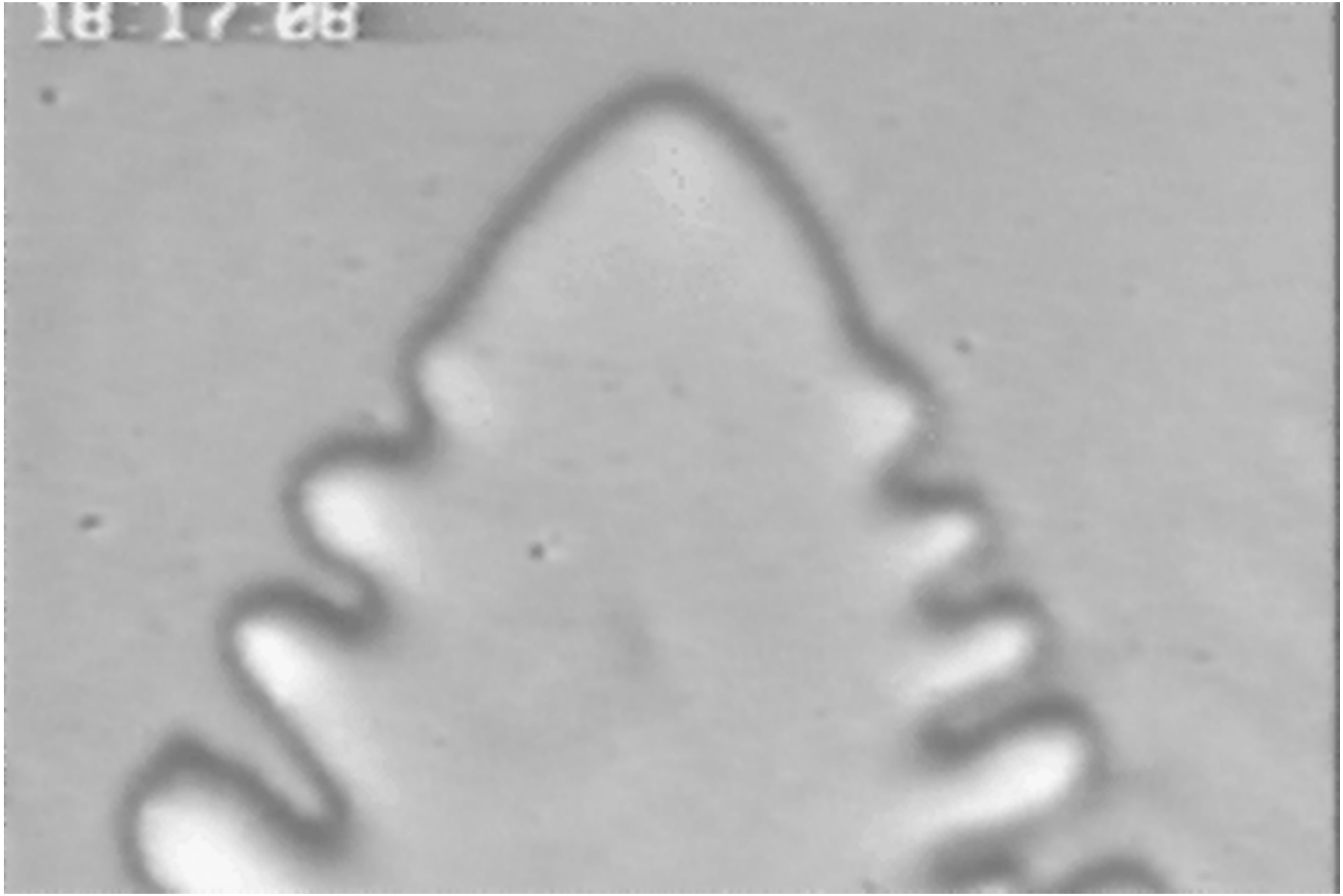
No convection

Visualisation in situ and in real time

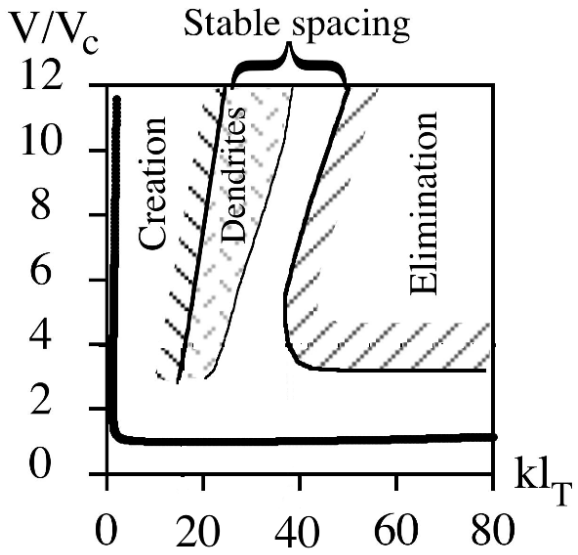
Planar instability and microstructures

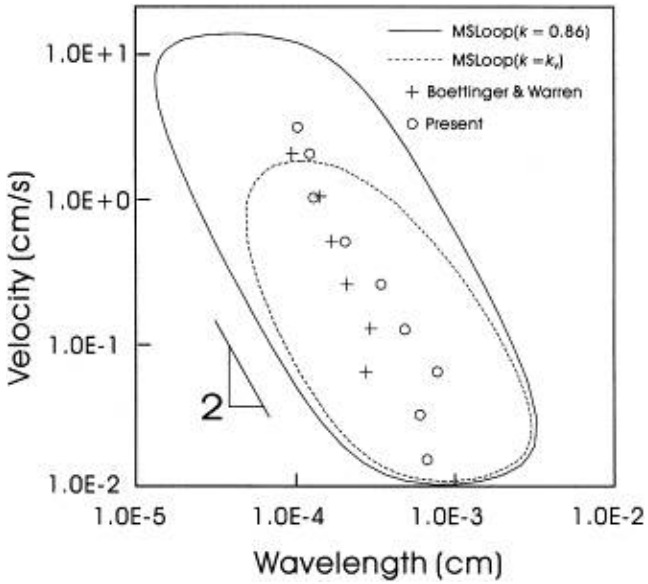


dendrite

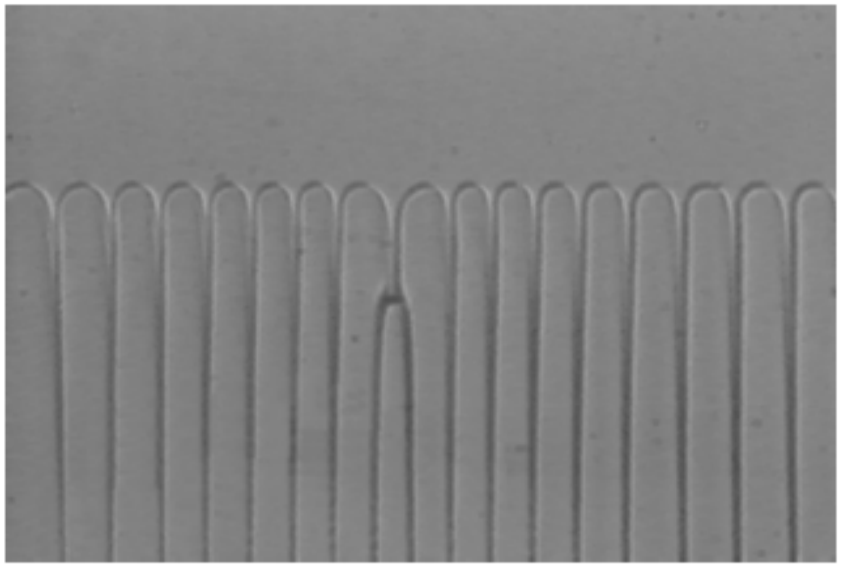


Secondary instabilities

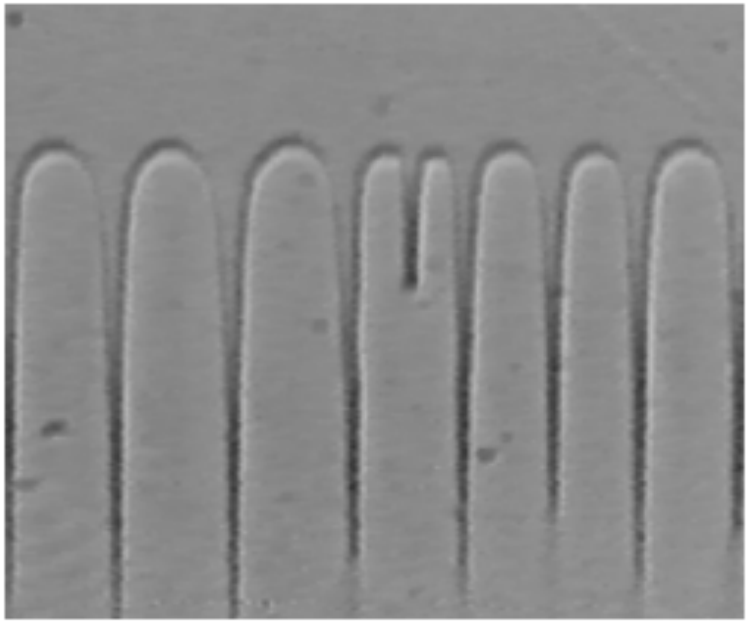




Elimination



Creation



Anisotropy

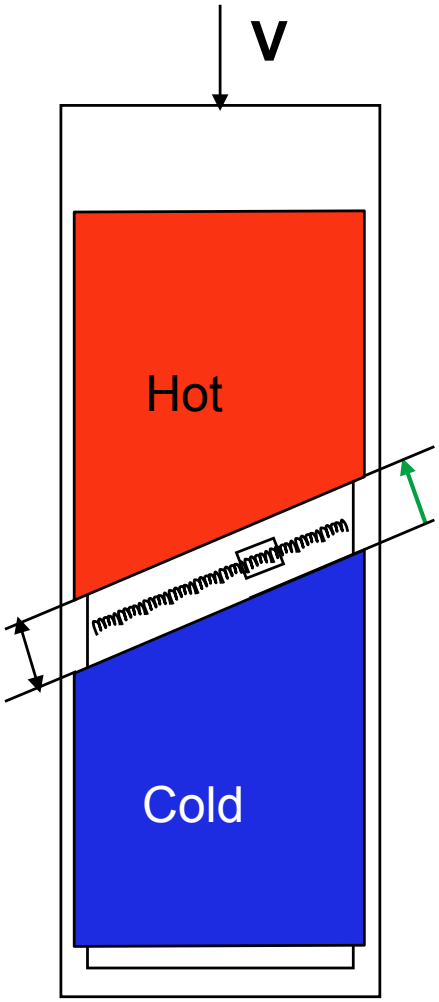
dependence of surface tension γ on crystal orientation θ

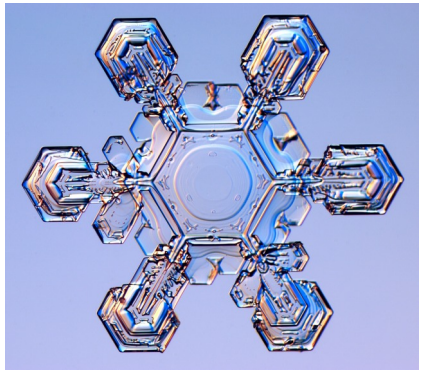
$$T = T_I - mc_I - T_M \frac{\gamma(\theta)}{Q}\kappa$$

$\gamma(\theta)/Q$: 10^{-3}

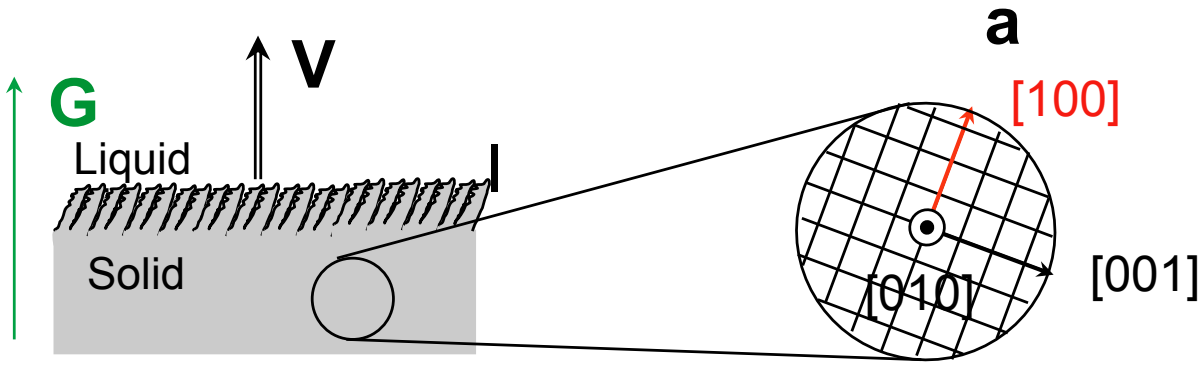
κ: 10^{-2}

implication on forms

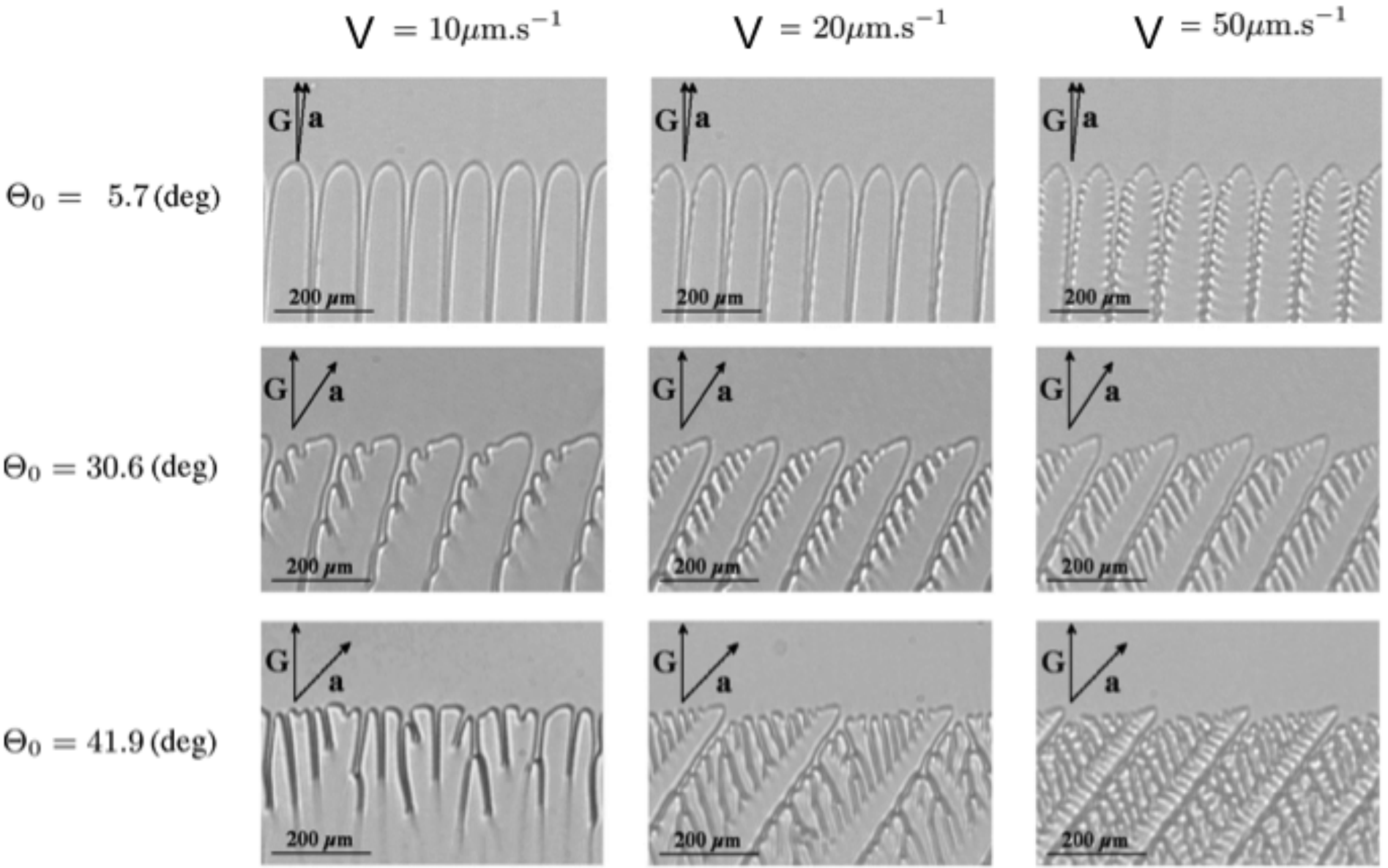




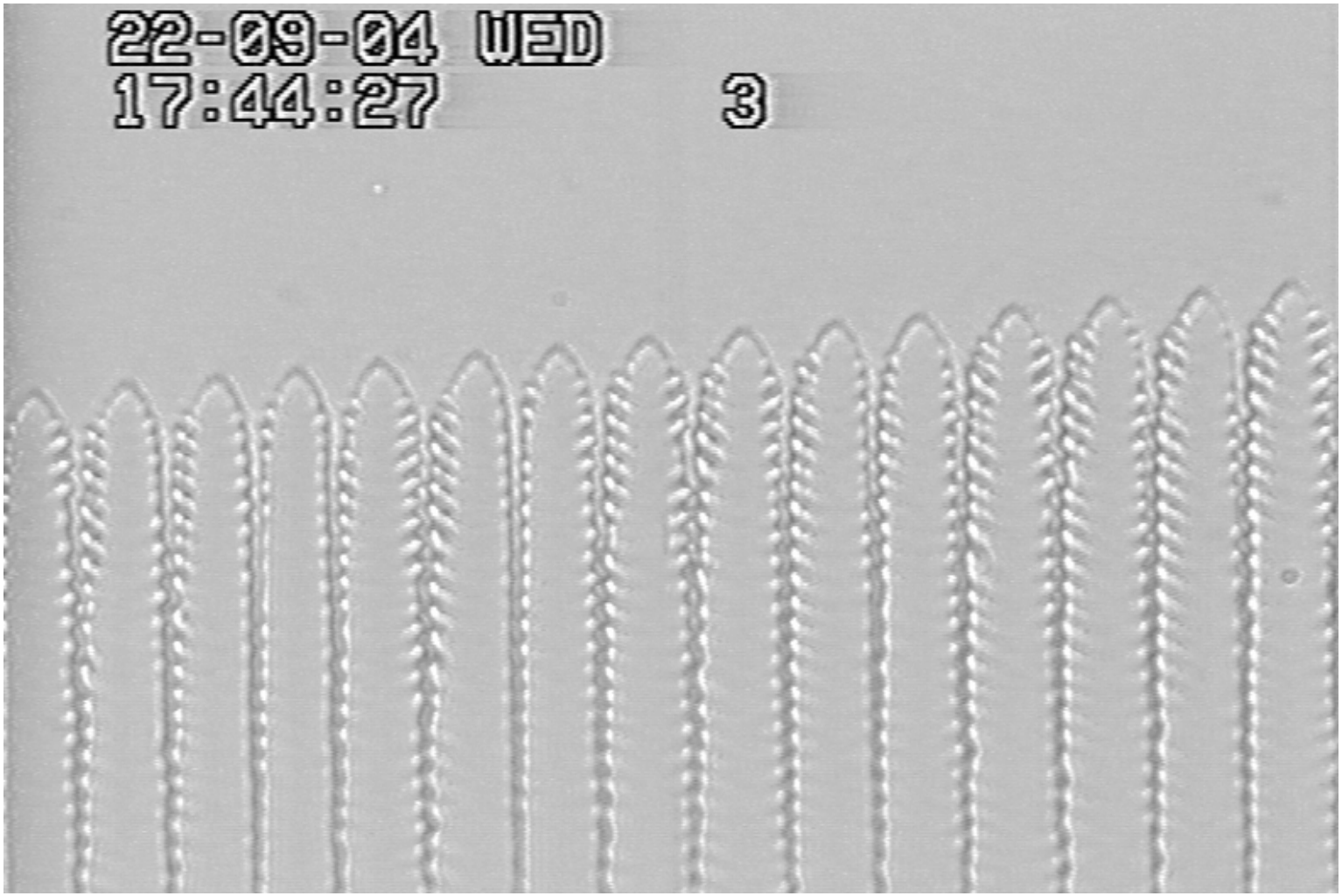
Snow crystal, K.Libbrecht



Morphologies



Deschamps, Georgelin, Pocheau PRE78,011605,2008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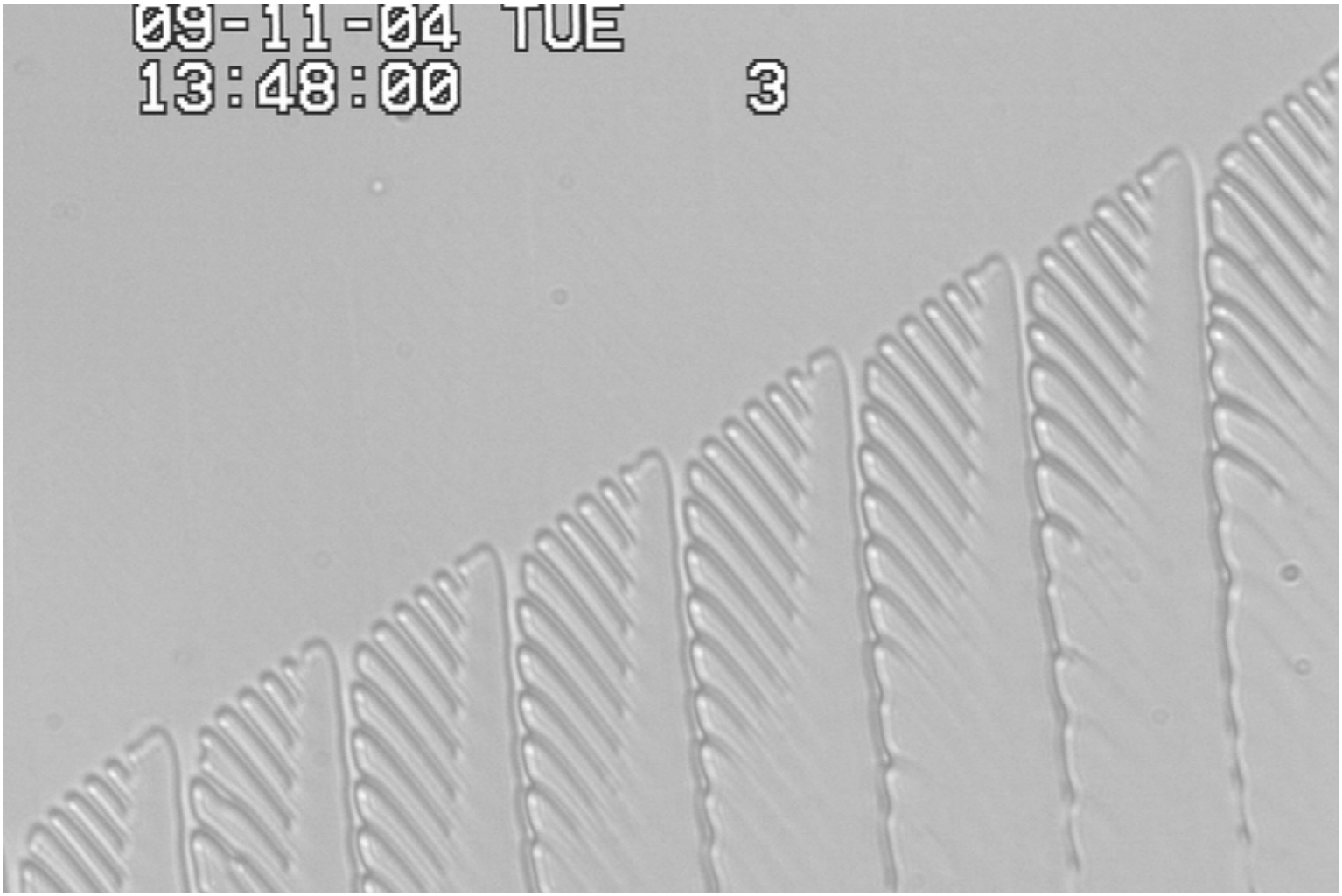
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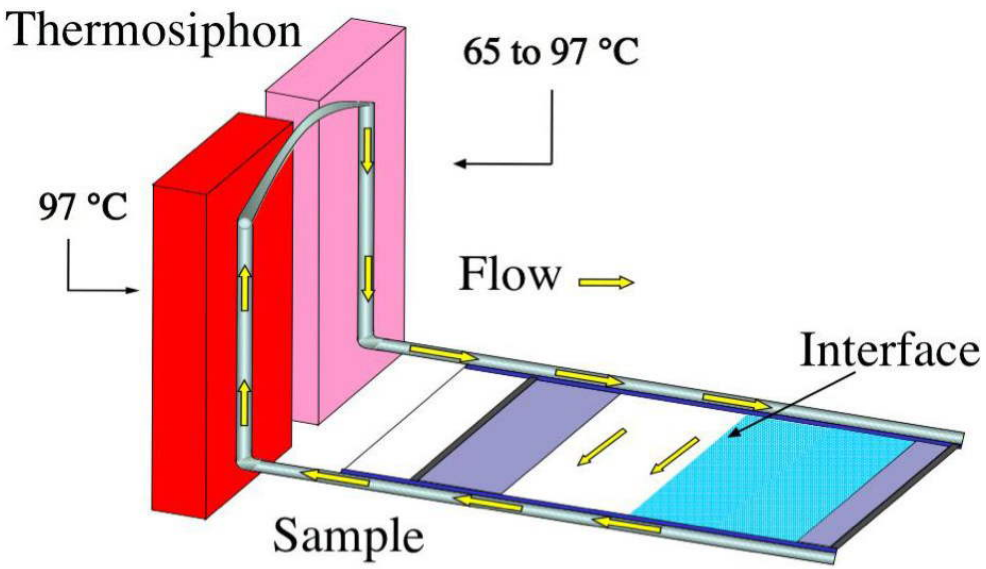
Flow on interfaces

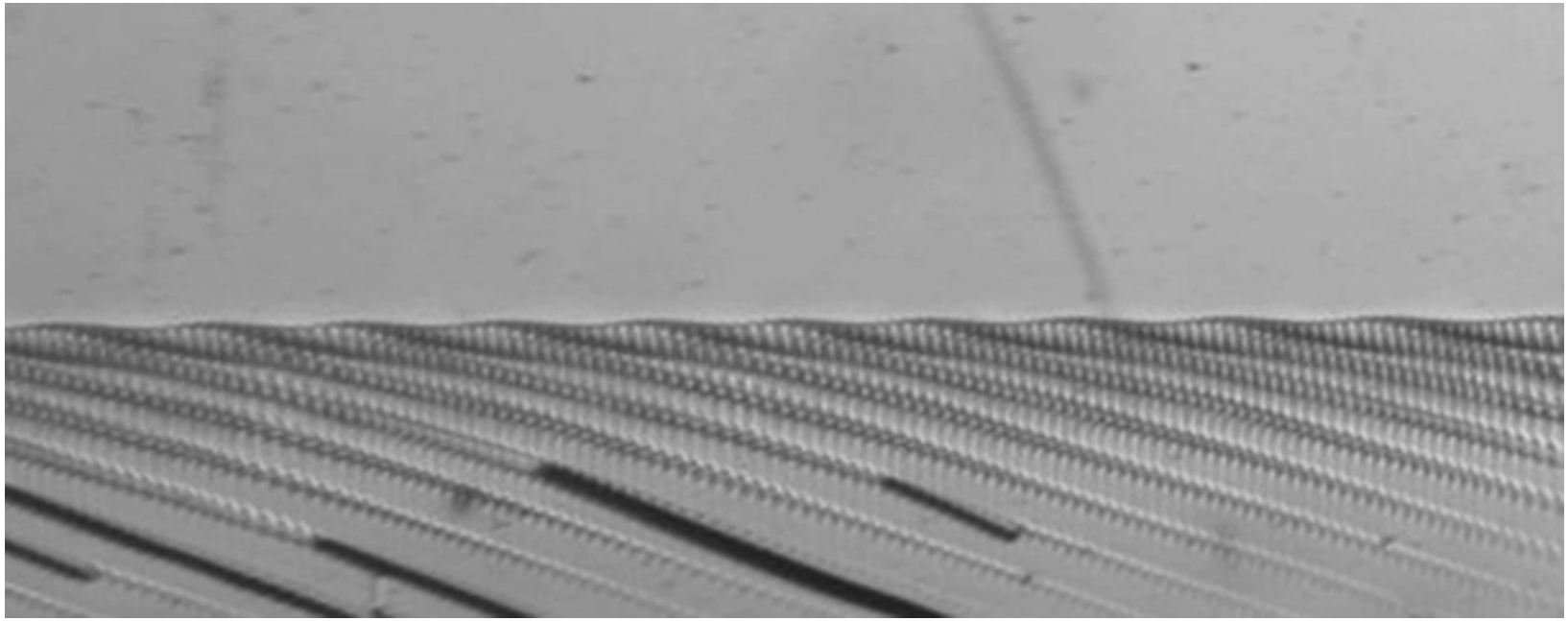
thermal gradient → convection

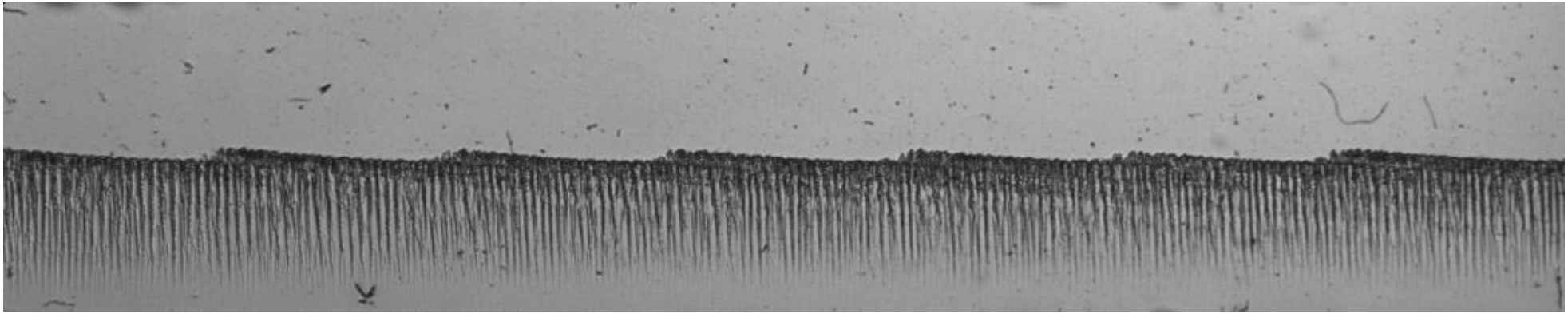
→ waves

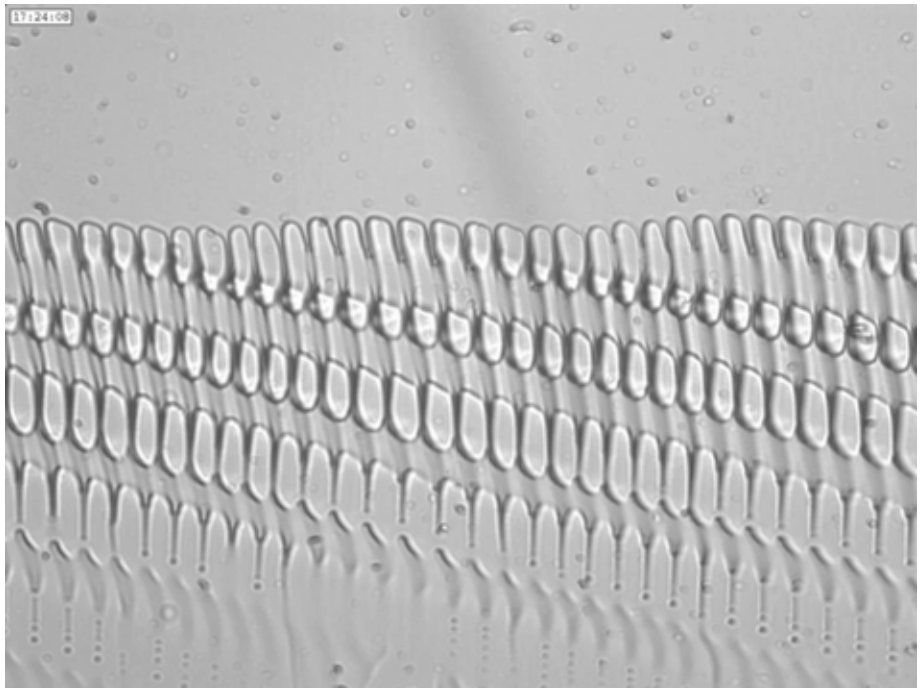
→ banding

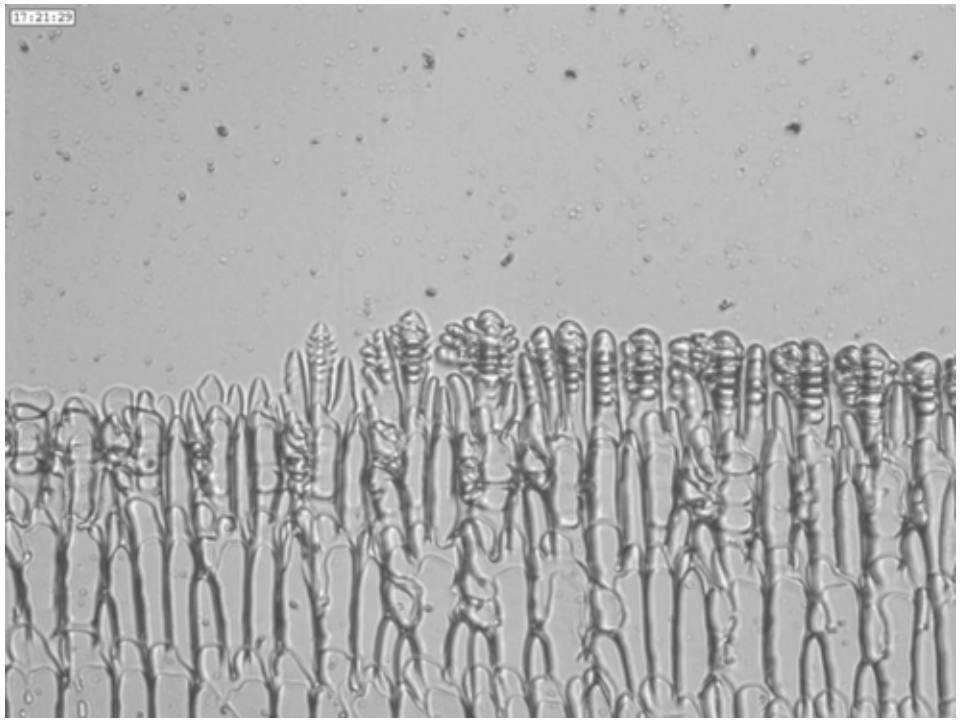
convection : external source (thermosiphon)







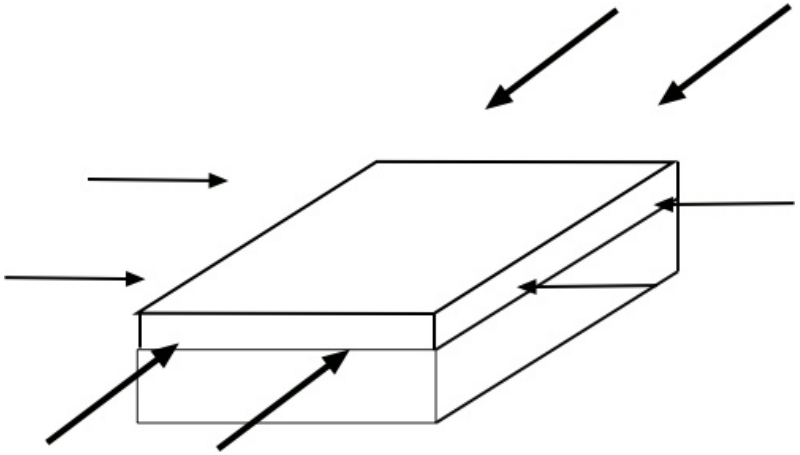




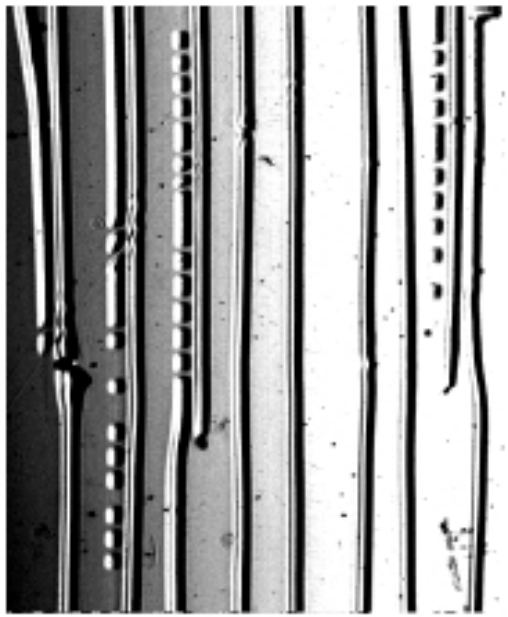
S/S interface

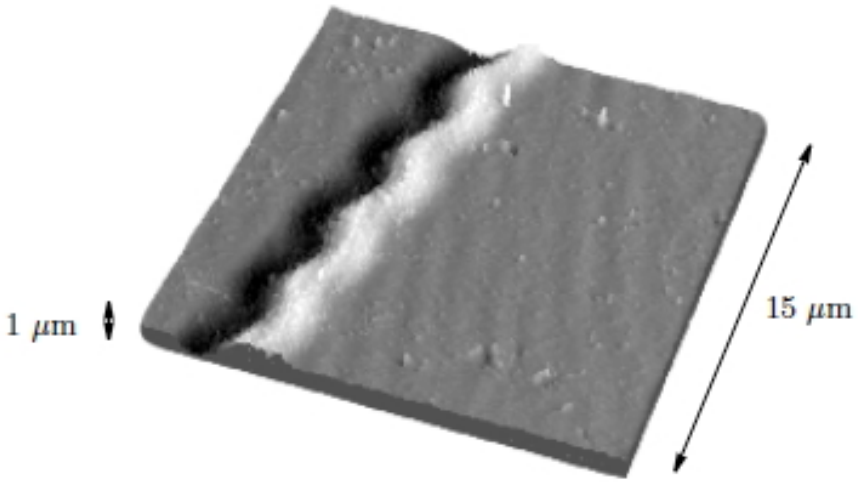
bi-layer

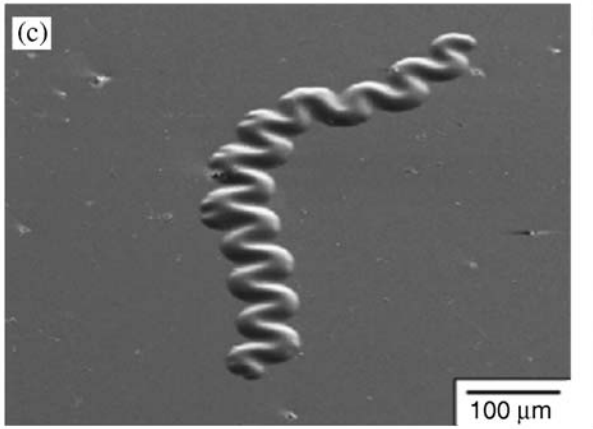
bi-axial compression



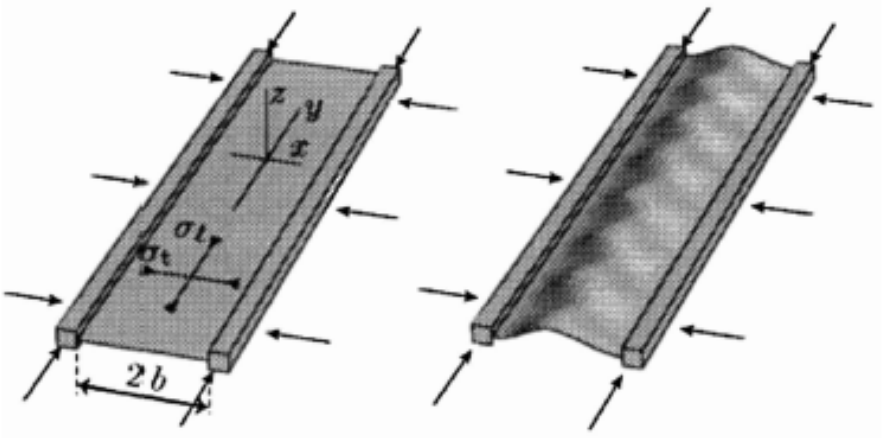
delamination patterns

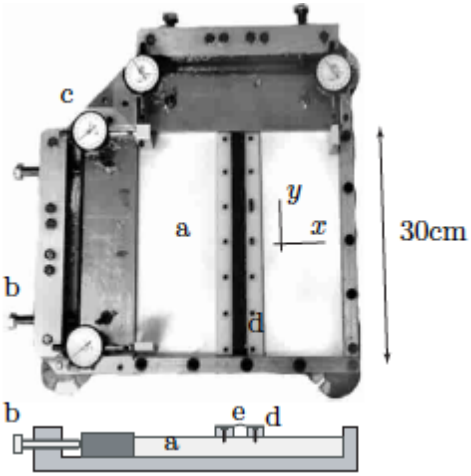




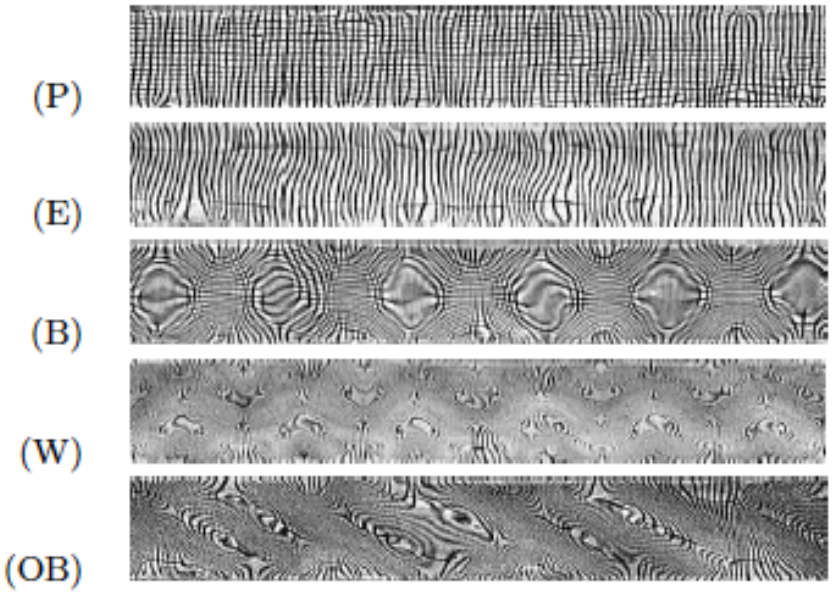


Model : bi-axially stressed sheet





Grid reflection : modes of distortion



B.Audoly et al., Eur. Phys. J. B **27**, 7, 2002

stress $\underline{\sigma}$ deflection ζ Airy potential χ

Föppl-Von Karman equations

$$D\,\Delta^2\zeta - h\left(\frac{\partial^2\chi}{\partial x^2}\frac{\partial^2\zeta}{\partial y^2} + \frac{\partial^2\chi}{\partial y^2}\frac{\partial^2\zeta}{\partial x^2} - 2\,\frac{\partial^2\chi}{\partial x\partial y}\frac{\partial^2\zeta}{\partial x\partial y}\right) = 0$$

$$\Delta^2\chi + E\left\{\frac{\partial^2\zeta}{\partial x^2}\frac{\partial^2\zeta}{\partial y^2} - \left(\frac{\partial^2\zeta}{\partial x\partial y}\right)^2\right\} = 0$$

$$\sigma_{ij} = (-1)^{i+j}\partial^2\chi/\partial x_i\partial x_j$$

Elastic energy

$$e_s = \frac{h}{2}\frac{E}{(1-\nu^2)}\{[Tr(\underline{\gamma})]^2 - 2(1-\nu)Det(\underline{\gamma})\}$$

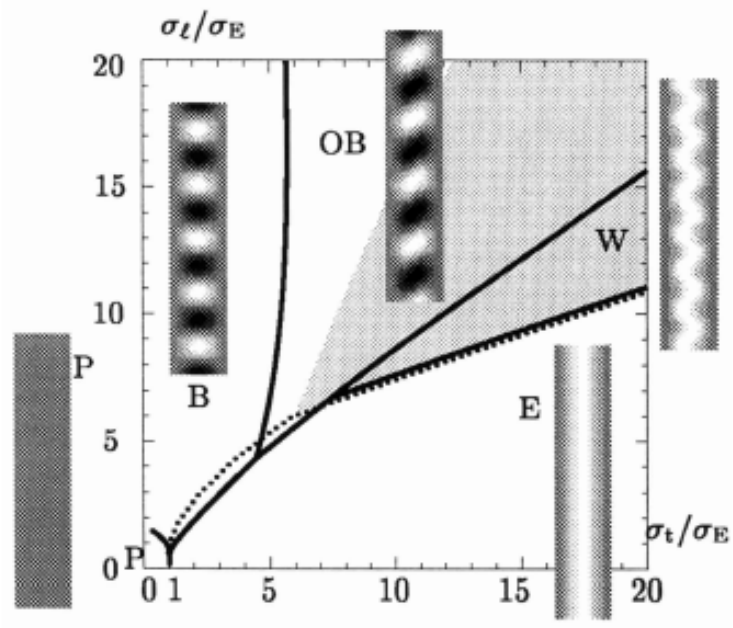
$$e_b = \frac{h^3}{24}\frac{E}{(1-\nu^2)}\{[Tr(\underline{C})]^2 - 2(1-\nu)Det(\underline{C})\}$$

$$e = e_s + e_b$$

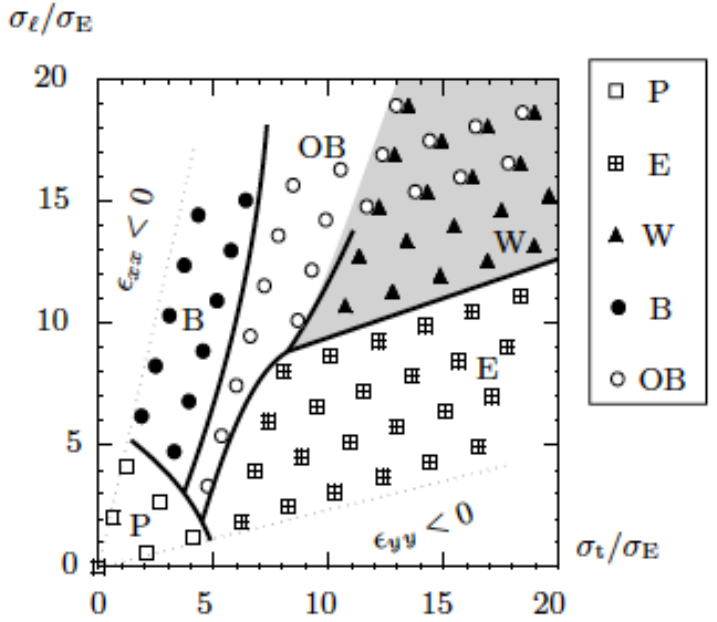
$\underline{\gamma}$ stretch

$\underline{C}$ curvature

solutions of elastic equations = minimization of elastic energy



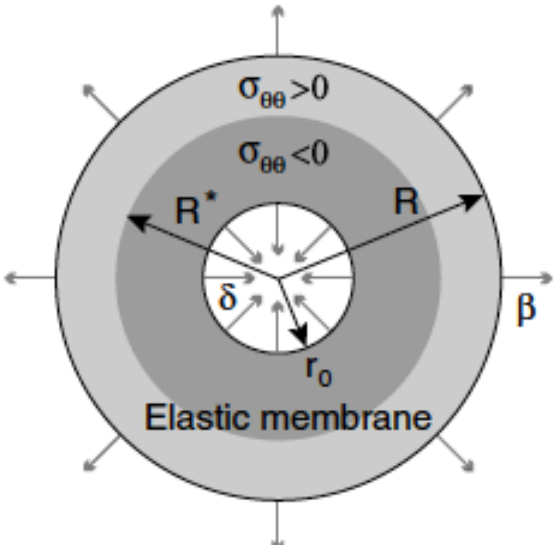
Experimental

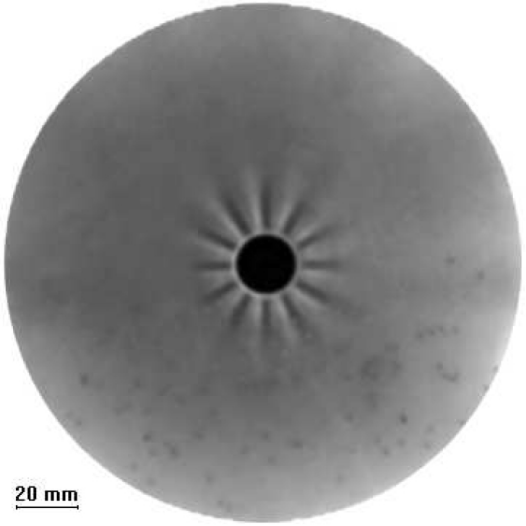


Solution of elastic equations

oblique modes OB = coupling between bumps (B) and worms (W)

Buckling by axial compression





minimalization of energy → selection of the number of wrinkles

J-C.Géminard et al., Eur. Phys. J. E 15, 117, 126 (2004).

Conclusion

- breaking of basic state / uniformity : instabilities

- time scales : linear instabilities

- length scales : secondary instabilities / non-linearities

- dependence on gradients, surface tension, anisotropy ...

- universal instabilities, possibly coupled

- possible restabilization : return to uniformity

- large background and solid framework to address pattern formation